

**T H E
P R A C T I C A L M E A S U R E R,
O R
Y O U T H ' S I N S T R U C T O R :**

C O N T A I N I N G

Instructions for measuring all Kinds of Superficies and Solids, by Feet and Inches and by Decimals; including the Manner of measuring the several Artificers Works relative to building; with a Sketch of the Bricklayer's and Carpenter's Dimension-Books, and the Manner of squaring, abstracting, and making out the Bills.

**W I T H E V E R Y O T H E R P A R T I C U L A R
N E C E S S A R Y F O R T H E I N F O R M A T I O N**

O F

T H E W O R K M A N , T H E B U I L D E R , &c.

To which are added,

Plain and easy Rules for measuring Land, Gauging, &c.

By **W I L L I A M P A I N ,**
Author of the **B U I L D E R ' S G O L D E N R U L E ,** and other Works.

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L I T T L E R E D - L I O N C O U R T , C H A R T E R - H O U S E L A N E ,
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P R I C E T W O S H I L L I N G S , S E W E D .

FRAGMENTS OF AN



OF THE

CONTAINING

Instructions for the use of the various kinds of Superintendence and Society, for the
and the various kinds of Superintendence and Society, for the
the various kinds of Superintendence and Society, for the
of the various kinds of Superintendence and Society, for the
Manner of Superintendence and Society, for the

WITH EVERY OTHER PARTICULAR

NECESSARY FOR THE INFORMATION

THE WORKMAN, THE BUILDER, &c.

To which are added

Plans and other Notes for the use of the Builder, &c.

BY WILLIAM PAIN,

Author of the Builder's Golden Rule, and other Works.

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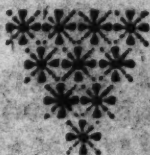
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M. DCC. LXXXII.

PRICE TWO SHILLINGS, SEWED.

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TABLE OF LAND MEASURE.

- 160 Square poles, or 4 roods, 1 acre of land.
- 40 Square poles 1 rood.
- 4840 Square yards, or 43560 square feet, 1 acre of land.
- 30 $\frac{1}{2}$ Square yards 1 square pole.
- 40 Poles in length and 1 in breadth 1 quarter of an acre.
- 160 Poles in length and 1 in breadth 1 acre.
- 80 Poles in length and 2 in breadth 1 acre.
- 40 Poles in length and 4 in breadth 1 acre.

TABLE OF LONG MEASURE.

Of Furlongs, Chains, and Poles.

A furlong is a length containing 10 chains or 40 statute poles, and is one eighth part of a mile: it is also called an acre's length, and the length of one chain is called an acre's breadth: for a piece of land, 10 chains in length and 1 chain in breadth, is equal to 160 square poles, or 1 statute acre.

Of Leagues, Miles, Degrees, Furlongs, Poles, Yards, and Feet.

20 Leagues, or 60 miles, 1 degree.

Note. A degree is the great measure of length, of which 360 are said to be the circumference of the earth.

In a mile are 8 furlongs, 320 poles, 1760 yards, 5280 feet, 63360 inches.

12 Inches 1 foot.

3 Feet 1 yard.

5 Yards and $\frac{1}{2}$ one pole.

6 Feet 1 fathom or 2 yards.

40 Poles, or 220 yards 1 furlong.

8 Furlongs 1 mile.

3 Miles 1 league.

TABLE OF SQUARE MEASURE.

144 Square inches 1 square foot.

46 Square quarters 1 square inch.

36 Square inches 1 quarter of a square foot.

Note. The square foot is divided in another manner, according to the rules of cross-multiplication, that is, into 12 parts, each part being 12 inches long and 1 inch in breadth: these are called long inches, and each part is equal to one 12th part of a square foot, or 12 square inches.

Of Square Yards.

A square yard is a square space, each side equal to 3 feet, and the whole containing 9 square feet. In a square yard there are 1296 square inches.

Of square Work, such as Flooring, Roofing, Tiling, &c.

A square is a square space containing 100 square feet; or it is a square figure whose sides are equal to 10 feet each way.

Of a square Pole.

A square pole is a square space containing 16 feet and a half each way, and therein are contained 272 feet one fourth; but the quarter of a foot is generally omitted, in business, and the rod is taken at 272 feet. 204 Feet is 3 quarters, 136 feet is the half, and 68 feet is the quarter.

OF AVOIRDUPOIS WEIGHT.

By avoirdupois weight all kinds of heavy commodities are bought and sold, as brails, copper, lead, grocery, &c. The smallest denomination is a dram.

16 Drams make 1 ounce.	28 Pounds a quarter of a hundred
16 Ounces 1 pound.	14 Pounds the 8th of a hundred.
112 Pounds 1 hundred weight.	7 Pounds the 16th of a hundred.
56 Pounds half a hundred.	

OF TROY-WEIGHT.

24 Grains 1 pennyweight.	100 Pounds 1 hundred weight.
20 Pennyweights 1 ounce.	50 Pounds half a hundred.
12 Ounces 1 pound.	25 Pounds a quarter of a hund.

By this weight, jewels, gold, silver, corn, bread, and liquids, are weighed.

OF BEER-MEASURE.

282 Solid inches 1 gallon of beer.	18 Gallons 1 kilderkin.
70½ Solid inches 1 quart.	36 Gallons 1 barrel.
35¼ Solid inches 1 pint.	54 Gallons 1 hoghead of beer.
9 Gallons 1 firkin.	

OF WINE-MEASURE.

231 Solid inches 1 gallon.	63 Gallons 1 hoghead.
57½ Solid inches 1 quart.	2 Hogheads 1 pipe.
28½ Solid inches 1 pint.	12 Pipes 1 tun of wine.



THE PRACTICAL MEASURER, &c.

A Table for finding the Area of Polygons.

MULTIPLY the square of the sides by the tabular numbers, and the product is the area of the polygon.

Number of Sides.	Names.	Multipliers.
3	trigon	.433013
4	tetragon	1.000000
5	pentagon	1.720477
6	hexagon	2.598076
7	heptagon	3.633959
8	octagon	4.828427
9	enneagon	6.181827
10	decagon	7.694209
11	endecagon	8.514250
12	dodecagon	9.330125

A Table of Twelves, to be learnt by Heart.

12	Inches one Foot,
12	Parts one Inch,
12	Seconds one Part,
12	Thirds one Second,
12	Fourths one Third,
12	Fifths one Fourth,
12	Sixths one Fifth,
12	Sevenths one Sixth, &c.

This is of great use in the multiplication of feet, inches, and parts.

As whole numbers increase in a ten-fold proportion to the left-hand, so, on the contrary, decimals decrease, to the

the right-hand, in a ten-fold proportion, as in the scheme annexed.

7 Tens of Millions,
6 Millions,
5 Hundreds of Thousands,
4 Tens of Thousands,
3 Thousands,
2 Hundreds,
1 Tens,
0 Units.
1 Tenth Part,
2 Hundredeth Part,
3 Thousandth Part,
4 Ten Thousandth Part,
5 Hundred Thousandth Part,
6 Millionth Part.

Inches reduced to Decimals of a Foot.		Inches and Quarters, of an Inch reduced to the Decimal of a Foot.			
Inches.	Decimals.	Inches.	Decimals.	Inches.	Decimals.
11	.9166	11 $\frac{1}{4}$	.9791	5 $\frac{1}{4}$	.4791
10	.8333	11 $\frac{1}{2}$	.9583	5 $\frac{1}{2}$	.4583
9	.75	11 $\frac{3}{4}$	.9375	5 $\frac{3}{4}$	.4375
8	.6666	10 $\frac{1}{4}$	.8958	4 $\frac{1}{4}$	.3958
7	.5833	10 $\frac{1}{2}$	.875	4 $\frac{1}{2}$	.375
6	.5	10 $\frac{3}{4}$	.854	4 $\frac{3}{4}$	.3541
5	.4166	9 $\frac{1}{4}$	.8125	3 $\frac{1}{4}$	.3125
4	.3333	9 $\frac{1}{2}$	.7916	3 $\frac{1}{2}$	.2916
3	.25	9 $\frac{3}{4}$	.7708	3 $\frac{3}{4}$	.2708
2	.1666	8 $\frac{1}{4}$	.7291	2 $\frac{1}{4}$	.2294
1	.0833	8 $\frac{1}{2}$	.7183	2 $\frac{1}{2}$	.2083
		8 $\frac{3}{4}$	.6875	2 $\frac{3}{4}$	.1875
		7 $\frac{1}{4}$	.6458	1 $\frac{3}{4}$	.1458
		7 $\frac{1}{2}$	.625	1 $\frac{1}{2}$	.125
		7 $\frac{3}{4}$	.6141	1 $\frac{1}{4}$	.1041
		6 $\frac{1}{4}$	.565	$\frac{3}{4}$	.0625
		6 $\frac{1}{2}$	.5416	$\frac{1}{2}$	.04166
		6 $\frac{3}{4}$	.5208	$\frac{1}{4}$	.02083

This table shews the decimal of any number of inches, from 1 inch to 11 inches.

This table shews the decimal of any number of inches and quarters, from $\frac{1}{4}$ of an inch to 11 $\frac{3}{4}$.

REDUC-



REDUCTION OF DECIMALS.

To reduce a vulgar fraction to a decimal is nothing more than to add cyphers, at discretion, to the numerator, and divide by the denominator.

To reduce two-thirds of a foot, or 8 inches, to the decimal of a foot. $3)2.0000$

.6666 decimal.

To reduce 9d. to the decimal of a pound.

$240)90000(.0375$

72

180

168

120

120

To reduce $\frac{3}{4}$ to the decimal of a pound.

$288)30000(.3125$

288

120

96

240

192

480

480

To find the Decimal of any Number of Shillings under 20.

To find the decimal of 18s. add a cipher and take half. 180 shillings

90 decimal of 18s.

To find the decimal of 15s. add a cipher and take half 150 shillings

75 decimal sought.

And so for any number of shillings under 20.

To find the value of any decimal fraction, in the known parts of the integer, of coin, weight, and measure.

What is the value of .578696 of a pound sterling?
Answer 11s. 6½d.

.578696
20 shillings

11.573920
12

6.887040
4

3.548160

What is the value of .625 of a pound troy weight?

.625 of a pound troy weight
12 ounces 1 pound troy

7.500

20 pennyweights 1 ounce

10.000

Answer, 7oz. 10dwts.

What is the value of .6725 parts of a hundred weight, or 112 pounds?

.6725

4 quarters in one hundred

2.6900

28 pound one quarter

55200

13800

19.3200

16

19200

3200

5.1200

Answer, 2qrs. 19lb. 5oz.

What

What is the value of .8958 parts of a foot?

.8958
12 inches.

10.7496

4

2.9984

Answer, 10 inches 2 quarters.

What is the value of .6125 parts of a yard?

.6125
3 feet

1.8375

12 inches

10.0490

Answer, 1 foot 10 inches.

ADDITION OF DECIMALS.

In addition of decimals, take care to keep units under units in whole numbers, and tenths under tenths in decimals.

Let it be required to add .7125 parts, and .42 parts together.

.7125
.42

1.1325

20

2.6500

12

7.8000

4

3.2000

When the decimals are added together, so many must be cut off as that decimal number consists of which contains the most places in your example; the rest, if any to the left-hand, are whole numbers, as in this example.

Answer, 1l. 2s. 7½d.

Let

Let it be required to add the following sums together.

$$\begin{array}{r}
 .7426 \\
 24.846 \\
 3.612 \\
 7.3 \\
 \hline
 36.5006
 \end{array}$$

Answer, 36l. 5-tenths or 10s.

If the numbers given to be added are not of the same denomination, they must be brought into the same as in the following example.

Let it be required to add .725 parts of a pound and .625 parts of a shilling into one sum; first reduce the fraction of a shilling to the fraction of a pound, which is done by annexing a cipher, then take half the sum, which is the decimal of a pound.

$$\begin{array}{r}
 .0625 \\
 .03125 \text{ half sum} \\
 \hline
 .09375
 \end{array}
 \quad
 \begin{array}{r}
 .725 \\
 .03125 \\
 \hline
 .75625
 \end{array}$$

.75625 equal 15s. 12d.

Let it be required to add .9166 of a foot, and .8125 of a foot, together.

$$\begin{array}{r}
 .9166 \\
 .8125 \\
 \hline
 1.7291
 \end{array}
 \quad
 \begin{array}{r}
 \text{feet and decimal parts.} \\
 36.5813 \\
 18.875 \\
 \hline
 55.4583
 \end{array}$$

1.7291 equal 1 foot 8 $\frac{3}{4}$ inches.

Answer, 55 feet 5 $\frac{1}{4}$ inches.

Subtraction in decimals differs but little from that of whole numbers, only mind to keep units under units in whole numbers, and tens under tens in decimals.

Let it be required to subtract .617 from .84125.

$$\begin{array}{r}
 .84125 \\
 .61700 \\
 \hline
 .22425
 \end{array}
 \quad
 \text{remainder, equal 4s. 5 $\frac{1}{4}$ d.}$$

If

If the decimal places in either of the numbers have less places one than the other, it must be made equal by adding cyphers.

Let it be required to subtract 6.9845 from 25.75, the work will stand thus :

$$\begin{array}{r} 25.7500 \\ - 6.9845 \\ \hline \end{array}$$

18.7655 remainder, equal 187. 147 3 1/2

Note, all fractions must be brought to the same denomination before the work can be done.

MULTIPLICATION OF DECIMALS.

Multiplication of decimals is the same as whole numbers, only, when the work is finished, you must take care to point off as many places of decimals in the product as there are places of decimals in the multiplier and multiplicand; but in case of want in the product, add cyphers to the left-hand, as in the following examples.

In multiplication of decimals it will be convenient to make that number the multiplicand that contains the most places of figures, though sometimes it may be less in quantity.

Note, if both terms to be multiplied be decimals, the product will be a decimal; or, if both be mixed, that is, if both consist of whole numbers and decimals, the product will be mixed; but if one be mixed and the other decimal, the product will be sometimes mixed and sometimes a decimal.

Let 5.125 be multiplied by 2.65

$$\begin{array}{r} 5.125 \\ 2.65 \\ \hline 25625 \\ 30750 \\ 10250 \\ \hline 13.58125 \end{array}$$

Let 69.25 be multiplied by .459

$$\begin{array}{r} 69.25 \\ .459 \\ \hline 62325 \\ 34625 \\ 27700 \\ \hline 31.78575 \end{array}$$

285.25	23564	.001472
445	.0126	.1045
142625	441384	007360
114100	147128	005888
114100	73564	0014720
126.93625	.009269064	.000538240

The above examples are given for practice.

DIVISION OF DECIMALS.

Division of decimals is performed after the same manner as division of whole numbers, the only difficulty is to know the value of the quotient, for resolving which, observe the following rule.

When the work of division be ended, count how many places of decimals there are in the dividend more than in the divisor, for that excess is the number of places to be separated in the quotient for decimals: But, if there be not so many places of figures in the quotient as the said excess, that deficiency must be supplied with cyphers in the quotient to the left-hand, with a point before them, so shall you discover the value of the quotient. If the divisor consist of more places than the dividend, there must be cyphers added to the dividend, to make it consist of as many or more places of decimals than the divisor; for the cyphers added must be reckoned as decimals, and, if any remain, more cyphers may be added, and the quotient carried to as many places of decimals as you please. The foregoing, well observed, will make the practice plain and easy.

A Table of the Divisors.

1	172
2	344
3	516
4	688
5	860
6	1032
7	1204
8	1376
9	1548

Let

Let it be required to divide 56 by 172.

$$172 \overline{) 56.0000(3255}$$

$$\begin{array}{r} 516 \\ \hline 440 \\ 344 \\ \hline 960 \\ 860 \\ \hline 1000 \\ 860 \\ \hline 140 \end{array}$$

The excess of decimals in the dividend above the divisor is 4, so, when the division is ended, there must be 4 places in the quotient, as in the work.

Let it be required to divide 72.1564 by .1347.

$$.1347 \overline{) 72.1564(535.68}$$

$$6735$$

Roots	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
Squares	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400	441	484	529	576	625	676	729	784	841	900	961	1024	1089	1156	1225	1296	1369	1444	1521	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801	10000
Cubes	1	8	27	64	125	216	343	512	729	1000	1367	1848	2464	3225	4140	5209	6442	7849	9440	11225	13216	15423	17856	20525	23440	26611	30048	33761	37750	42025	46588	51449	56618	62105	67920	74073	80575	87436	94667	102278	110279	118680	127491	136722	146383	156484	167035	178046	189527	201488	213939	226890	240351	254332	268843	283894	299495	315656	332387	349698	367599	386000	404911	424342	444303	464804	485855	507466	529647	552408	575759	599710	624271	649452	675263	701714	728825	756506	784767	813618	843069	873130	903811	935132	967103	1000000														

Let 489 be a number, and let the square root thereof be required.

First, you will point given number, that is, make a point over the unit place, and another on the hundreds, and so on every second figure throughout, then seek the greatest number in the first column to the left hand, placing the square number under the first figure, and finding, so, the remainder being down.

In this work the excess is nothing, but I have added two ciphers and carried the quotient to two places of decimals.

Let 489 be a number, and let the square root thereof be required. First, you will point given number, that is, make a point over the unit place, and another on the hundreds, and so on every second figure throughout, then seek the greatest number in the first column to the left hand, placing the square number under the first figure, and finding, so, the remainder being down.

457) 1250000 (2.735

2014

3360

3199

1610

13700

2390

2285

105

In this example the excess is 3, therefore there must be 3 decimals in the quotient, as per work.

EXTRACTION OF THE SQUARE ROOT.

A Table of Squares and Cubes, with their Roots.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

Let 4489 be a number given, and let the square root thereof be required.

First, you must point the given number, that is, make a point over the units place, and another on the hundreds, and so on every second figure throughout, then seek the greatest square number in the first point to the left-hand, placing the square number under the first point, and subtract; and to the remainder bring down the next point, and call that the resolvend; then double the quotient, and place it for a divisor on the left-hand of the resolvend, and seek how often the divisor is contained in the resolvend, reserving always the units place, and put the answer in the quotient, and also on the right-hand side of the divisor; then multiply by the figure last put in the quotient, and subtract the product from the resolvend, as in common division, and bring down

down the next point to the remainder, if there be any more, and proceed as before, till the whole is done. — See the work.

$$\begin{array}{r} 4489(67 \\ 36 \\ \hline 127)889 \\ 889 \\ \hline \end{array} \qquad \begin{array}{r} 67 \\ 67 \\ \hline 469 \\ 402 \\ \hline 4489 \text{ Proof.} \end{array}$$

Note, if the given number be mixed, consisting of whole numbers and decimals together, make the number of decimal places even, that is, 2, 4, 6, 8, &c. that there may be a point fall on the units place of the whole numbers, and by this method the square root of any number may be found near the truth.

Let 106929 be a number given, and let the square root thereof be required.

$$\begin{array}{r} 106929(327 \text{ root} \\ 9 \\ \hline 62)169 \\ 124 \\ \hline 647)4529 \\ 4529 \\ \hline \end{array}$$

To find the square root of a fraction, if it be a vulgar fraction, it must be reduced to a decimal, and pointed off as in whole numbers.

Let $\frac{1}{4}$ be a vulgar fraction given, then reduce it to a decimal as follows :

$$\begin{array}{r} 4)3000 \\ \hline .7500(866 \text{ root} \\ 64 \\ \hline 166)1100 \\ 986 \\ \hline 114)10400 \\ 10356 \\ \hline \end{array}$$

Let $\frac{1}{2}$ be a vulgar fraction given, whose square root is required.

$$8)7000$$

$$875000(.9375 \text{ root})$$

$$81$$

$$183)650$$

$$549$$

$$1865)10100$$

$$9125$$

$$775$$

Let .125 be a decimal fraction, to find the square root.

$$125000(.353)$$

$$9$$

$$65)350$$

$$325$$

$$703)2500$$

$$2109$$

$$391$$

Let it be required to find the square root of .00715.
In this case a cipher must be added to the right-hand, to make the places even.

$$.007150(.084)$$

$$64$$

$$164)750$$

$$656$$

$$.94$$

The above examples will be sufficient for practice, as they will be repeated in the following work.

MEN-

15. 6. 0

10. 3

155. 0. 6

3. 10. 0

158. 10. 6

1941

13. 2. 10

1990

18.4 half base line

16.6 perpendicular

152-207,107 01

081-2-24-25, 25, 25

1951

[illegible]

5.4 5.4

02. 6 . 60

1914

To work this example, first, multiply 16 ft by 18; then, for the 6 inches, take half the top-line, which is 9 feet 2 inches; and for the 4 inches take one third of 16 feet, which is 5 feet 4 inches; add all up, and the sum will be 102 feet 6 inches.

Suppose

Suppose the whole base line to be multiplied by the whole perpendicular, the superficial content will be double the area of the triangle. See the work.

36 . 8

36 . 6

216

36 . 8

36 . 6

36 . 4

5 . 2

5 . 4

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

To work this example, multiply 16 feet by 36; then, for the 6 inches, take half the top-line, which is 18 feet 4 inches; then, for the 8 inches take one third of 16 ft and let it down twice; or two-thirds may be taken at once, which is 10 feet 8 inches; add all up, the sum is 605 feet 0 inc. the half of which is equal the area of the triangle, as before.

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0

605 . 0 area of the triangle, as before.

To find the superficial content of the trapezium *b d a c*, *fig. 3, plate A*, multiply half the base-line *a b*, by the two perpendiculars *e, d*, and *f, c*, the product will be the superficial of the trapezium *b, d, a, c*, the base line *a, b*, 25 feet 6 inches, the perpendiculars 8 feet 3 inches each.

25 . 6 whole base line *a, b*,

16 . 6 two perpendiculars *e, f, d*

150

25

12 . 9

8 . 0

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

420 . 9

Multiply 26 by 25, then take half 25 ft 6 inc. which is 12 ft 9 inc. and half 16 ft is 8 feet; add all up, the sum is 420 ft 9 inches.

420 . 9 area of the parallelogram

210 . 4 . 6 area of trapezium, as before.

12 . 9 half base line

16 . 6 two perpendiculars

72

12

6 . 4 . 6

8 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

4 . 0 . 0

Multiply 12 by 16; and the half of 12 ft 9 inc. is 6 ft 4 inc. 6 parts; the half of 16 is 8, and half of 8 is 4, or 3-4ths of 16 is 12.

210 . 4 . 6 area of the trapezium

Fig. 4, plate A. To measure irregular figures which have more sides than 4, and the sides and angles unequal, all such figures must be divided into trapeziums and triangles, and the areas added together; so will you have the area of the whole figure. The base line *a, b*, is 25 feet 6 inches, the perpendiculars *f, d*, 6 feet 4 inches, the perpendiculars *b, i*, 4 feet, the perpendiculars *e, h*, 5 feet, and *g, d*, 9 feet 6 inches, the base line *a, c*, 28 feet 4 inches, the superficial content is required.

14 . 6 two perpendiculars a, b, c .

54 . 2 half base line a, b, c .

56

14

2 . 5

7

two

205 . 5

To work this example, multiply 14 by 14, and the product is 196. Then take the sixth part of 14 feet 6 in. and find it is 2 feet 5 in. take half 14 feet; and the sum is 215 feet 5 inches.

25 . 6 base line a, b, c .

100 . 4 two perpendiculars a, b, c .

250

8 . 6

3

2631 . 6

131 . 9

205 . 5

337 . 2

area of trapezium, a, b, c, d, e, f .

area of trapezium, a, b, c, d, e, f .

area of the whole figure.

Fig. 5, plate A. Of regular polygons. These are such as have more sides than four; all the sides and angles thereof being equal. Let a, b, c, d, e, f , be a regular hexagon, each side thereof being 12 feet 6 inches and the perpendicular g, h , 10 feet 3 inches; add all the sides together and the sum is 75 feet, the half thereof is 37 feet 6 inches, which multiply by 10 feet 3 inches, the perpendicular g, h . The product will be the superficial content.

75 . 0 . 0 sum of the sides

5 . 1 . 6 half perpendicular of g, h

375

6 . 3

3 . 1 . 6

384 . 4 . 6

area, as before.

37 . 6 half sum of the sides

10 . 3 whole perpendicular

370

9 . 4 . 6

5

384 . 4 . 6

area of the hexagon

For the 3 in. take a 4th part of the top-line, which is 9 ft 4 in. 6 parts; and for the 6 inches take half 10 feet, which is 5 feet.

Every

Every regular polygon is equal to the parallelogram or long square, whose length is equal to half the sum of the sides, and the breadth equal to the perpendicular of the polygon, as appears by the foregoing figure; for the parallelogram r, f, t, u , is made of six equilateral triangles; and the parallelogram r, f, t, u , is composed of six of those triangles, that is, five whole ones and two halves, therefore it is equal in area to the hexagon.

Fig. 6, plate A. Of a circle. To find the area or superficial content of the circle, multiply half the diameter by half the circumference, and the product is the area or superficial content. Suppose the diameter of the circle, *fig. 6*, to be 22.6 , then the circumference will be 71 , the half is 35.5 , and the half of 22.6 is 11.3 ; these multiplied together, the product will be the superficial content. To find the side of a square that will be equal in area to the circle, is nothing more than to extract the square root out of the circle's area, and the root is the side of the square required.

$ \begin{array}{r} 35.5 \\ 11.3 \\ \hline 106.5 \\ 355 \\ 355 \\ \hline 401.15 \end{array} $	$ \begin{array}{r} 401.15 (20.0287 \text{ sides of the square } a, b, \\ 4 \\ \hline 4002011500 \\ 8004 \\ \hline 40048349600 \\ 320384 \\ \hline 4005672921600 \\ 2803969 \\ \hline 117631 \\ \hline 20.0287 \\ 20.0287 \\ \hline 1402009 \\ 1602296 \\ 400574 \\ 40057400 \\ \hline 401.14882369 \text{ area of the circle as before.} \end{array} $
---	--

By

Fig. 6, plate A.

By having the diameter, to find the side of the square that is equal in area to that circle.

If the diameter be 1, the side of the square equal will be .8862, therefore as 1 : .8862 :: 22.6, the diameter to the side required.

$$1 : .8862 :: 22.6$$

22.6

53172

17724

17724

20.02812 side of the square *a, b*.

Fig. 6, plate A. By having the circumference, to find the side of the square.

If the circumference of a circle be 1, the side of the square equal will be .2821, therefore as 1 : .2821 :: 71 the circumference.

$$1 : .2821 :: 71$$

71

2821

19747

20.0291 side of the square *a, b*, as before.

Fig. 6, plate A. Having the diameter, to find the side of a square which may be inscribed in that circle.

If the diameter be 1, the side of the square inscribed will be .7071, therefore as 1 : .7071 :: 22.6

$$1 : .7071 :: 22.6$$

22.6

22426

24242

24242

15.98046 side *c, d*, inscribed.

Fig. 6, plate A. Having the circumference, to find the side of a square which may be inscribed.

If the circumference be 1, the side of the square inscribed will be .2251, therefore as 1 : .2251 :: 71.

$$\begin{array}{r} 1 : .2251 :: 71 \\ \hline .2251 \\ 15757 \\ \hline 15.9821 \text{ side of the square } c, f, \end{array}$$

Or, if you square the semi-diameter, and double that square, the square root of the double square will be the side of the square inscribed.

$$\begin{array}{r} 14.3 \text{ semi-diameter} \\ \hline 11.3 \\ \hline 339 \\ 113 \\ \hline 113 \\ \hline 127.69 \text{ the square of } c, o, \text{ which doubled, because } c, o, \\ 2 \text{ is equal } o, f. \end{array}$$

255.38 (15.98 root which is the side of the square c, f , as before.

$$\begin{array}{r} 255.38 \\ \hline 125 \end{array}$$

Fig. 6, plate A. Having the diameter, to find the side of a square which may be inscribed in the circle.

$$\begin{array}{r} 31.88 \text{ diameter} \\ \hline 25504 \\ \hline 196 \end{array}$$

Having the diameter of a circle to find the circumference. As 7 is to 22, so is the diameter to the circumference, that is, if the diameter of any circle be multiplied by 22, and that product divided by 7, the quotient

quotient will be the circumference; or by Vanculen's proportion, multiply the diameter by 3.141593, and the product will be the circumference: or if 355 be multiplied by the diameter of any circle, and that product divided by 113, the quotient will be the circumference.

If the circumference be given to find the diameter of any circle, then as 22 is to 7, so is the circumference to the diameter: or multiply the circumference by .318309, and the product will be the diameter of the circle: or if the circumferences be multiplied by 113, and that product be divided by 355, the quotient will be the diameter of the circle:

$$\begin{array}{r} 22.6 \\ 22 \\ \hline \end{array}$$

$$\begin{array}{r} 452 \\ 452 \\ \hline \end{array}$$

$$7)4972($$

$$71.02 \text{ circumference}$$

$$22)497.14$$

$$22.6 \text{ diameter}$$

$$\begin{array}{r} 355 \\ 22.6 \\ \hline \end{array}$$

$$\begin{array}{r} 2130 \\ 710 \\ \hline \end{array}$$

$$113$$

$$113)80230(71 \text{ circumference.}$$

$$791$$

$$113$$

$$113)791$$

$$355)8023(22.6 \text{ diameter}$$

$$710$$

$$923$$

$$710$$

$$2130$$

$$2130$$

$$7$$

3.141593 circumference of the unit; but
 22.6 in most cases 3.1416, is sufficient.

$\begin{array}{r} .18849558 \\ 6283186 \\ \hline 6283186 \end{array}$

71.0099018 circumferences.

.318309 diameter of the unit, or one.

$\begin{array}{r} 6390891162 \\ 21302970540 \\ \hline 5680792144 \\ 710099018 \\ \hline 2130297054 \end{array}$
 22.6030908320562 diameter.

Let it be required to find the area of a circle whose diameter is 1, or unit. The rule is, to multiply half the diameter by half the circumference 3.141593, the diameter 1, or unit 1; .5 half the diameter, or 5-tenths.

$\begin{array}{r} 3.141593 \\ 1.5707965 \\ \hline \end{array}$

1.5707965 half circumference.

.5 half the diameter.

.78549825 area of the circle whose diamet. is 1.

Suppose the area be required when the circumference is 1. When the circumference is 1, the diameter will be .318309. Multiply half the diameter by half the circumference, and the product will be the answer.

$\begin{array}{r} .318309 \\ \hline \end{array}$

.159154 half the diameter.

.5-10ths, half the circumference.

.795770 area of a circle whose circumference is 1.

If the diameter of any circle be squared, and the product be multiplied by 7854, the last product will be the superficial content of the circle.

If the circumference of any circle be squared, and that product be multiplied by .79577, the last product will be the superficial content of the circle.

Fig. 7, pl. A. Of a semicircle. To find the superficial content, multiply half the arch line, by the semidiameter, the product will be the superficial content.

Suppose the whole diameter a, b , to be 22.6, the half c, d , 11.3, which is the semi-diameter, and the circumference

ference is 71; a, d, b , the half is 35.5, which is the arch line of the semi circle; and the half of that is 17.75, to be multiplied by 11.3, and the product will be the superficial content.

17.75

11.3

5325

1775

1775

200.575

area.

Fig. 8, pl. A. To find the superficial content of a quadrant or fourth part of a circle, multiply half the arch-line of the quadrant 8.875, by the semi-diameter 11.3, and the product is the superficial content.

8.875

11.3

26625

8875

8875

100.2875

area.

Fig. 9, pl. A. Suppose it is required to find the length of the arch-line of a segment geometrically, divide the base-line $a b$ into 4 parts, and turn one up to d , then draw the line $c d$, which is half the arch of the segment $a e b$.

Fig. 10, pl. A. To find the superficial content of the sector a, b, c, d , multiply half the arch line by the semi-diameter, and the product is the superficial content; the semi-diameter a, f , 24.6, and the arch line a, d, b , is 47; the half is 23.5, and that multiplied by 24.6, the product is the superficial content of the sector a, b, c, d . See the work.

74.6 semidiameter.

23.5 half arch.

1230

738

492

578.10 superficial content.

Fig. 10, plate A.

To find the length of the arch line, arithmetically, multiply the chord line, a, d , by 8, and from that product subtract 43, the chord of the whole segment a, b , and divide the remainder by 3, the quotient is the arch line a, d, b .

23 chord line b, d .

8

184

43

3)141(47 arch line, a, d, b .

23.5 half arch line.

Fig. 11, pl. A. To find the area or superficial content of the segment a, b, d , first find the center of the whole circle, which to do, square half the chord-line a, b , the product is 324, which divide by 6, the versed sine e, d , the sum will be 54, to which add 6, the versed e, d , the whole diameter of the circle, is 60, half is 30, equal the semi-diameter a, c , or b, c ; to find the arch line, square the semi-chord a, e , and versed-sine e, d , and add the two products together, and out of that sum extract the square root, which root is the chord line of the segment d, a ; then multiply the chord d, a , by 8, and from that product subtract the chord line a, b , and divide the remainder by 3, the quotient will be the arch line a, d, b ; then multiply half the arch line by the semi-diameter b, c , the product will be the superficial content of the sector a, b, c, d ; next find the superficial content of the triangle a, b, c , which to do, subtract d, e , the versed sine, from the

the semi-diameter, the remainder is the perpendicular of the triangle $\triangle$, which is to be multiplied by half the chord line a, b , the product will be the area of the triangle, which subtract from the area of the sector, the remainder will be the area of the segment a, d, b . See the work.

36 chord line a, b	30 semi-diameter b, r
18 half chord a, c	6 versed sine d, e
18	24 perpendicular a, e
144	18 half chord line a, b
18	192
16) 324	24
54	432 area of the triangle a, b, c
6 versed sine e, d	578.70 area of the whole sector a, b, r, d
60 whole diameter	432 area of the triangle
30 semi-diameter	146.70 area of segment

the chord line d, a 18.97	versed sine 6	semi-chord 18
8	6	18
151.76	36	144
36	324	18
3) 145.76	360 (18.97 chord line a, b)	3240
38.58 arch line a, d, b	1	
19.29 half arch line	28) 260	
30 semi-diameter	224	
578.70 area of the sector	369) 3600	
	3321	
	279	

Fig. 12, pl. A. To measure the sector $a b d$, which is greater than a semi-circle whose semi-diameter o, d , is 24.5,

24.5, which multiplied by half the arch line a, b, d , the product will be the area or superficial content.

45.6 arch line a, b

24.5 semi-diameter

2280

1824

912

1117.20 area or superficial content of the whole sector.

Fig. 13, pl. A. To find the superficial content of the segment $e f g$, which is greater than a semi-circle, the dimensions the same as the above sector, to be worked in the same manner, only observe to find the area of the triangle e, i, g, h , and add that to the area of sector, and the sum will be the area of the whole segment $e f g h$.

1117.20

270.6

1387.80 area of the whole segment.

12.3 perpendicular b, i

22 half base line e, g

246

246

270.6

To find the length of the arch line, see pages 21 and 22.

Fig. 8, pl. B. To find the superficial content of the oval, square the diameters and multiply the product by 7854, the product will be the superficial content.

20.5

15.2

410

1025

205

312.60

.7854

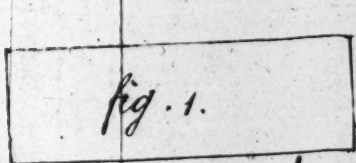
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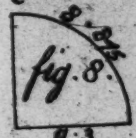
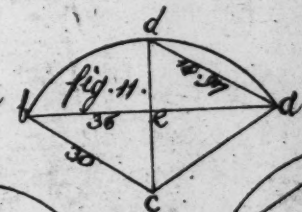
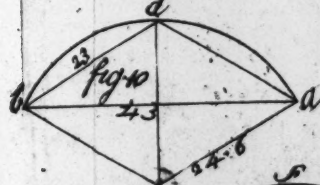
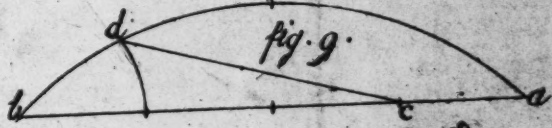
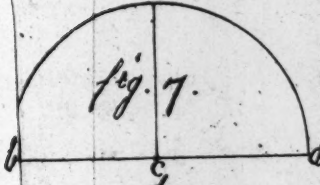
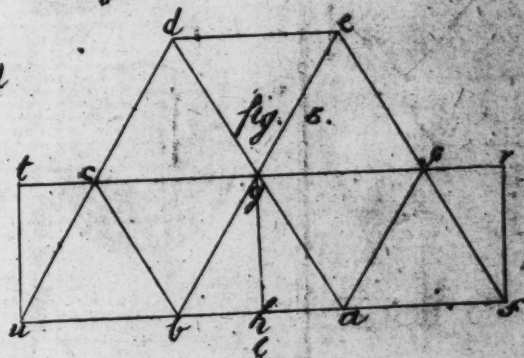
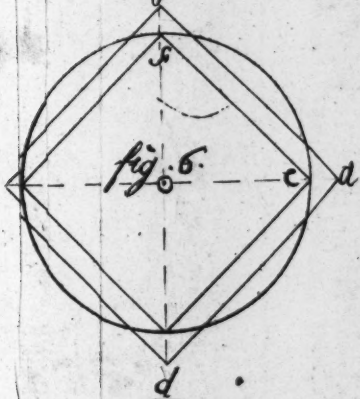
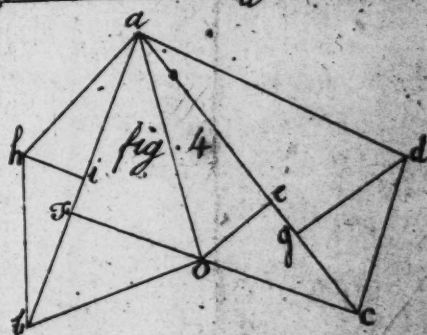
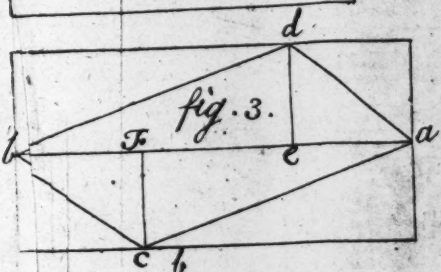
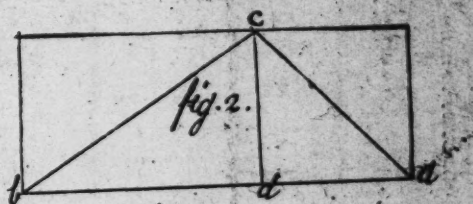
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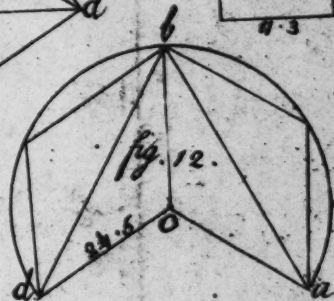
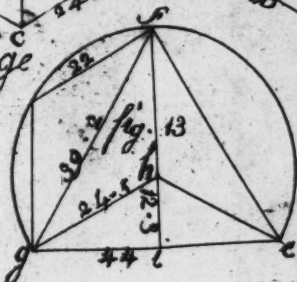
244.730640 superficial content.



PA



face Page
24



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MENSURATION OF SOLIDS.

Fig. 1, A. B. To find the solid content of the square pyramid *a, b, c*, whose base is 1 foot 3 inches, and the height 12 feet, multiply 1 foot 3 by 1 foot 3, and that product by one third of the height, will give the solid content.

$$\begin{array}{r}
 1.3 \\
 1.3 \\
 \hline
 1.3 \\
 3.9 \\
 \hline
 1.6.9 \\
 4 \text{ one third} \\
 \hline
 6.3.0 \text{ six feet } 3
 \end{array}$$

Every pyramid is one third of its circumscribing prism, that is, of equal base and altitude, what form soever its base is of, whether square or multangular.

Suppose the prism *a, b, d, e*, to be circumscribed about the pyramid *a, b, c*, and of equal base and length, if the length of the prism 12 feet, be multiplied into the area of the base, the solid content of the prism will be 18 feet, 9 inches, and one third of that is the solid content of the pyramid.

$$\begin{array}{r}
 1.3 \\
 1.3 \\
 \hline
 3.9 \\
 1.3 \\
 \hline
 1.6.9 \text{ area of the base.} \\
 12 \text{ length.} \\
 \hline
 12 \\
 6.9 \\
 \hline
 \frac{1}{3} 18.9 \\
 \hline
 6.3 \text{ solid content as before.}
 \end{array}$$

Fig. 2, pl. B. Suppose $abcd$ be the frustum of a square pyramid, the side of the greater base, 1 foot 6 inches, the side of the lesser base, 1 foot, and the height 18 feet, first multiply the two sides together, 1 foot 6 by 1 foot, the product is 1 foot 6, and the difference of the sides is 6, whose square is 36, a third part thereof is 12, or 1 long inch, which add to 1 foot 6 inches, the product is 1 foot 7, the area of a mean base, which multiply by 18 feet, the length, the product is 28 feet 6 inches, the solid content.

	f. in.	or thus
6	1.6	1.6
6	1.0	1.6
—	—	—
3) 36	product 1.6	0.0.1
—	add 0.1	1.6
12	—	6
—	—	—
	Multiply 1.7	3
	by 18.0 height	2.5
	18.0	1.6
	9.0	1.6
	11.6	1.6
Solid content in	—	4.9
feet and inches, 28.6	—	trip of a mean area.
	—	6
	—	1 of the height.
	—	28.6
	—	content as before

Fig. 3, pl. B. Suppose abc to be a triangular pyramid, each side of the base 15 inches, and the perpendicular 12 inches. Multiply half the base, 7.5, by 12, the perpendicular, the product is 90.0, the area of the base, which multiply by one third of the height, and the product will be the solid content. Whole height of the pyramid 9 feet. See the work.

1.3
6
—
7.6
3 one third
—
12) 22.6
—
1.10.6 solid
content.

Note,

Note, if the area of the base be multiplied by the whole height, that will be the solid content of the circumscribing prism, *abef*, one third of which is the solid content of the pyramid.

$$\begin{array}{r} 7.5 \\ 12 \\ \hline 150 \\ 75 \\ \hline 90.0 \\ 3 \text{ 1-third.} \\ \hline \end{array}$$

12) 270.0 (22.5 or
solid content 1.10,5
as before.

Suppose the pyramid *a, b, c*, to be cut off at *d*, then the bottom part *a, b*, will be the frustum of a triangular pyramid. To find the solid content, multiply 15 inches, the side of the greater base, by 6 inches, the side of the lesser base,

$$\begin{array}{r} 15 \text{ from great base} \quad 15 \text{ inches, side of great base} \\ 6 \text{ subtract lesser base} \quad 6 \text{ inches, side of lesser base} \\ \hline 9 \quad 90 \text{ product} \\ 9 \quad 27 \text{ add a third of differences} \\ \hline 3) 81 \text{ square of the difference} \quad 117 \text{ sum} \\ \hline 27 \text{ a third part} \quad 433 \text{ tabular number} \\ \hline 351 \\ 351 \\ \hline 468 \\ 50.661 \text{ mean area} \\ 6 \text{ feet, the height of frustum} \end{array}$$

$$12) 303.966$$

$$12) 25.335$$

2.111 solid content of frustum

Fig. 4, Plate B.

Suppose *a, b, c*, to be a hexangular pyramid, each side 18 inches, and the perpendicular 15 inches .5, that to be multiplied by half the sum of the sides 54 inches, and the product is 8370, which multiply by one third of the height 5 feet, the product will be 41850, which divide by 12, and that quotient by 12. The last quotient will be the solid content, 29.0625 feet.

(28)

15.5 perpendicular.

54 half sum of the sides.

620

775

837.0 area of the base.

5 a third part of the height.

12)4185.00(348.7500

29.0625 the solid content of the pyramid, equal to one third of the circumscribing prism.

Suppose the top of the hexangular pyramid to be cut off at d , then the bottom part will be the frustum of a hexangular pyramid, each side of the greater base being 18 inches, each side of the lesser base 9 inches, and the height 10 feet 5; the solid content is required. Multiply the greater side, 18, by the lesser side, 9, and the product will be 162; then the difference between 18 and 9 is 9, which, squared, makes 81, and a third thereof is 27, which added to 162, the sum is 189. Multiply this last number by the tabular number 2.5980, and the product is 491.022, the area of a mean base, which multiplied by the height, 10.5 feet, the product is 5155.7310. Then divide the last product by 12, and the quotient arising from that by 12. The last quotient will be the solid content of the frustum. See the work.

18	18	2.5980 tabular number.
9	9	189
162	9	233820
27 add	9	207840
189	3)81 square of the diff.	25980
	27 a third part.	491.0220
		10.5
		24551100
		49102200
		12)5155.73100
		12)429.64425
		Solid content of the frustum. 35.80368

Fig. 5.

Fig. 5 plate B. A cone is a solid, having a circular base, and growing smaller till it ends in a point, called the vertex. To find the solid content the rule is, multiply the area of the base by a third part of the height, and the product is the solid content. See the work.

18	7854	tabular number.	
18	324		
144	31416		
18	15708		
324	23562		
	254.4696		
		5.0 a third part of the height.	

12)1272.3480	106.02900	
	8.83575	solid content.

Note, the cone is one third of its circumscribing cylinder *abcf*; for if the solid content of the cone be multiplied by 3 the product will be the solid content of the cylinder.

Suppose the top-end of the cone to be cut off at *d*, the bottom part will be the frustum of a cone, which represents round timber-trees. When the top-end of it is cut off, the remaining part is the frustum of a cone. To find the solid content the rule is, to the rectangle of the greater and lesser diameters add one third part of the square of their differences, and multiply the sum by the tabular number .7854, the product will be a mean area, which multiply by the perpendicular height of the frustum, and the product will be the solid content.

18 greater diameter.	7854	18	
12 lesser diameter.	228	12	
216	62832	6	
12 add,	15708	6	
228	15708	3)36	square of the diff.
	12)179.0712		
		12 a third part.	
	12)14.9226	12435	mean area.
		9	height of the frustum.
	12)11.1915		
		9.3260	solid content.

A cylinder is a round solid, having its bases circular, equal, and parallel, in form of a roller. To find the solid content the rule is, multiply the area of the base by the length, and the product is the solid content. *abc* fig. 6, plate B, is a cylinder; whose diameter *ab* is 18 inches, and length 15 feet. First, square the diameter, 18 inches, and it makes 324, which multiply by the tabular number .7854, and the product will be 254.4696; then multiply that by the length, 15 feet, the product will be 3817.0448; divide that by 12, and the quotient arising therefrom by 12, and the last quotient is the solid content.

18	.7854 tabular number.	
18	324	
144	31416	
18	15708	
324 square of the diameter.	23562	
	254.4696 area.	
	15 length.	
	12723480	
	2544696	
	12)3817.0440	
	12)31.8087	
	26.5072	
	Solid content, 26 feet 5-10ths.	

Fig. 7. plate B. Of a Sphere, or Globe.

A sphere is a round body, every part of whose surface is equally distant from a point within it, called the center; and it may be conceived to be formed by the revolution of a circle round its diameter. To find the solid content, the rule is, multiply the axis, or diameter, into the circumference, and the product is the superficial content, which multiply by a sixth part of the diameter, and the product is the solid content. Or, multiply the whole

whole diameter by the superficial content, and a sixth part of that will be the solid content. Or, as 21 is to 11, so is the cube of the diameter to the solid content.

Let *e f*, fig. 7, plate B, be a globe, whose diameter is 30 inches; then the circumference will be 94.2480. Multiply the circumference by the diameter, and the product will be 2827.4400, which is the superficial content in inches; these being multiplied by a sixth part of the diameter, the product will be solid content.

94.2480 circumference.

30 diameter.

2827.4400 superficial content, in inches.

5 a sixth part of the diameter.

14137.2000 solid content, in inches.

These divided by 1728 give the solid content, in feet.

1728) 14137.2000 (8.18

13824

3132

1728

14040

13824

216

Note, If the diameter of a globe be 1, the solid content will be .5236, and if the circumference be 1, the solid content will be .016887.

To find the Solid Content of the Frustum of a Sphere.

Let *a b c d*, fig. 7, plate B, be the frustum of a sphere. Suppose *a b*, the diameter of the frustum's base, to be 18 inches, and *c d*, the height, 5 inches, the solid content is required. To three times the square of the semidiameter of the frustum's base add the square of the height of the frustum, and multiply that sum by the height, and when that product is multiplied by .5236, the product arising therefrom will be the solid content.

$$\begin{array}{r}
 5 \\
 5 \\
 \hline
 25 \text{ square of } c d. \quad 209440 \\
 15708 \\
 5236 \\
 \hline
 \text{Solid content of} \\
 \text{the frust. in inches. } 701.6240
 \end{array}$$

$$\begin{array}{r}
 9 \text{ semi-diameter of } a b. \\
 9 \\
 \hline
 81 \text{ square of ditto.} \\
 3 \\
 \hline
 243 \text{ three times the square.} \\
 25 \text{ add the square of } c d. \\
 \hline
 268 \quad [\text{height}] \\
 5 \text{ multiply by } c d, \text{ the} \\
 \hline
 1340
 \end{array}$$

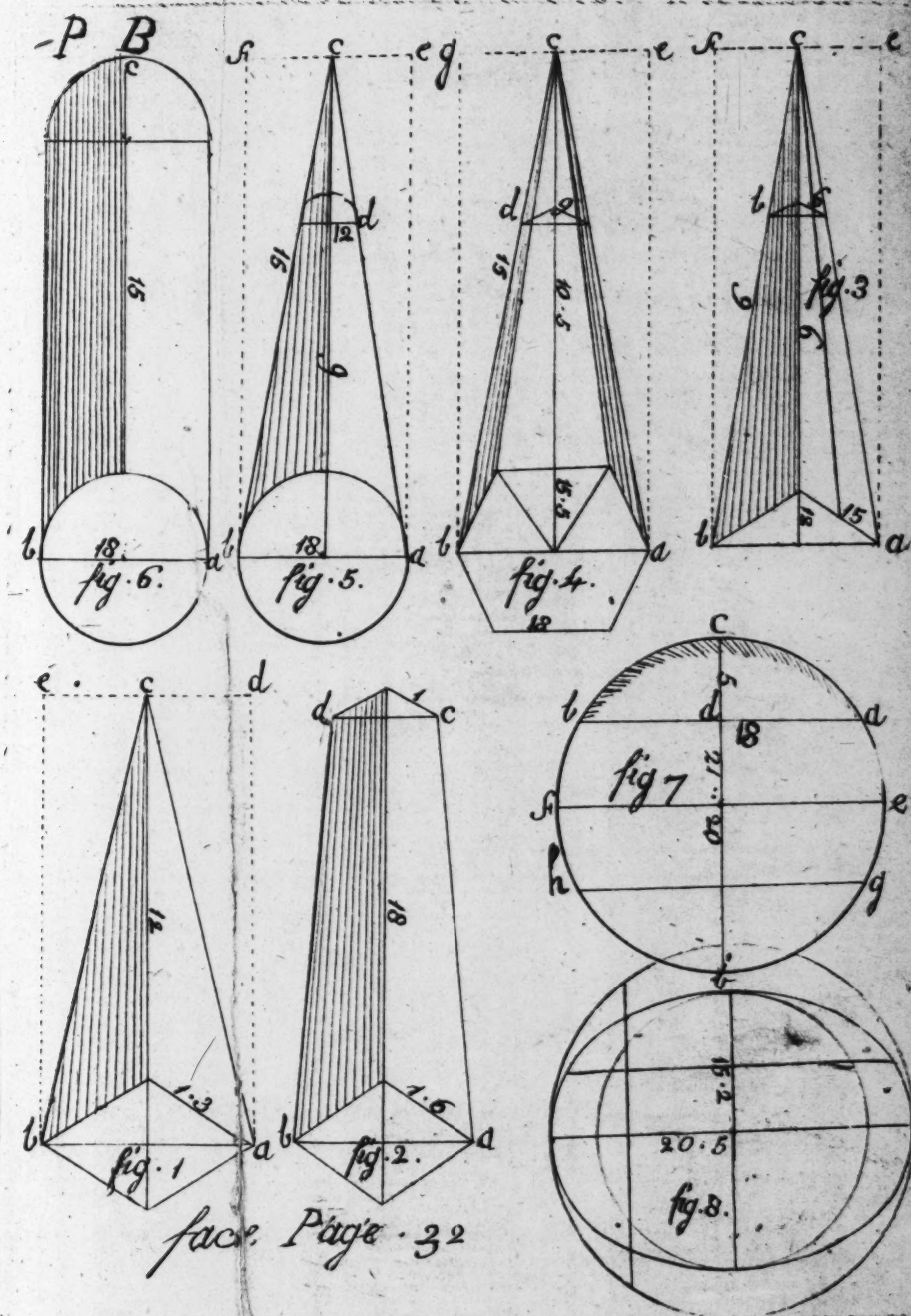
But if the diameter of the sphere and height of the frustum be given, to find the solid content, then, from three times the diameter subtract twice the height of the frustum, and multiply the remainder by the square of the height, and when that product is multiplied by 5236, this last product will be the solid content of the frustum. Suppose the diameter of the whole globe to be 21.20 inches.

$$\begin{array}{r}
 21.20 \\
 3 \\
 \hline
 63.60 \text{ three times the diameter.} \\
 10. \quad \text{twice the frustum's height subtract.} \\
 \hline
 53.60 \\
 25 \text{ square of } c d. \text{ Solid content in inches. } 701.624000 \\
 \hline
 26800 \\
 10720 \\
 \hline
 1340.00
 \end{array}$$

Suppose it be required to find the solid content of the middle part, $a b g b$, called the middle zone of a sphere. To twice the square of the diameter, $e f$, add the square of the diameter of the frustum's base, $a b$, and divide by 3.8197, then multiply the quotient by the height or thickness of the middle zone, and the product will be the solid content of the middle zone required. Or, if the solid content of the two frustums be subtracted from the content of the whole globe, the remainder will be the content of the middle zone.

Suppose

253



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Suppose fig. 8, plate B, to be a solid body. To find the solid content, square the conjugate diameter, 15.2, and multiply that product by the length or transverse diameter, 20.5, then multiply the product arising from that by 5236, the last product will be the solid content of the spheroid.

Mensuration of Square Timber Scantling.

Of Scantling cut fit for Building.

Suppose a piece of scantling 45 feet long, 4 by 3, how many cube feet? Mul-

$$\begin{array}{r} 45 \\ 3 \overline{) 12} 45 \\ \underline{12} \\ 0 \end{array}$$

tiply 3 by 4, which is 12 cubic inches, or one long inch. As 1 will neither multiply nor divide, take the twelves in 45, which is 3 feet 9 inches, the cube content.

Suppose a piece of scantling 65 feet long, 4 by 4. Four times 4 is 16, which is 1 long inch and 4 parts. As 1 does not multiply, put down 65 feet, the length, and for the 4 parts take one third of 65, which is 21.8, add all up together, and divide by 12, the quotient is 7 feet 2 inches 8 parts.

$$\begin{array}{r} 65 \\ 4 \overline{) 21} 8 \\ \underline{21} \\ 0 \end{array}$$

Suppose a piece of scantling 48 feet long, 5 by 4. Say 4 times 5 is 20, which is 1 long inch and 8 parts, or 12 cube inches and 8 parts. As 1 will not multiply, set down 48, and for the 8 inches take two-thirds of 48, which is 32. Set down and add up, the sum is 80. The twelves in 80 are 6 times and 8 over. So the cube content is 6 feet 8 inches, or two thirds of a foot.

$$\begin{array}{r} 48 \\ 5 \overline{) 20} 8 \\ \underline{20} \\ 0 \end{array}$$

Suppose a piece of scantling 32 feet long, six inches square, say 6 times 6 is 36, the twelves in that are 3, which multiply by 32, the length, and the product is 96; the twelves in it are 8, which is 8 feet, the cube content.

$$\begin{array}{r} 32 \\ 6 \overline{) 36} \\ \underline{36} \\ 0 \end{array}$$

Suppose a piece of scantling 54 feet long, 6 by 4. Say, 4 times 6 is 24, that is, two long inches, which multiplied by 54, the length, the product is 108; the twelves in that are 9, which is 9 cube feet, the content.

24
34
108
12
9

Suppose a piece of scantling 56 feet long, 9 inches by 6. Say, 6 times 9 is 54, which is 4 long inches and 6 parts, which multiplied by 56 feet, the length, the product is 252, being divided by 12, the content is 21 feet cube.

54
56
252
12
21

Suppose a piece of scantling 65 feet long, 12 by 9. Say, 9 times 12 is 108, the twelves in that are 9, which multiply by the length, 65, and the product is 585, the twelves in that 48 feet 9 inches, the cube content.

108
9
585
12
48
9

Suppose a piece of square timber to be 44 feet long, 1 foot 6 inches, by 1 foot 3 inches. First multiply by 3 inches, saying, 3 times 6 is 18, set down 6 parts and carry 1, saying, 3 times 1 is 3 and 1 I carry is 4 inches, set down 4 inches; then once 6 is 6, and once 1 is 1, set down and add all up, the product is 1 foot 10 inches 6 parts, to be multiplied by 44 feet, the length. Set down 44, then, for the 10 inches, take the half of 44, which is 22 feet, then

18
6
4
6
44
22
44
88
22
6

there is a third left in ten inches to be taken in 44, which is 14 feet 8 inches. to be set down, then, for the 6 parts, take the half of 44, which is, 22 inches, or 1 foot 10 inches; add all up together and the sum is 82 feet 6 inches.

Suppose a piece of timber 35 feet long, and 1 ft 3 in square. Here, instead of multiplying by 35, I multiply by 7 and 5, for 5 times 7 is 35, which shortens the work and is more easy. The content is 54 feet, 8 inches, 3 parts.

$$\begin{array}{r}
 100 \cdot 28 \\
 1 \cdot 3 \\
 1 \cdot 3 \\
 \hline
 48 \cdot 3 \cdot 9 \\
 10 \cdot 3 \\
 1 \cdot 6 \cdot 9 \\
 \hline
 002 \cdot 28 \\
 2 \cdot 10 \cdot 11 \cdot 3 \\
 \hline
 54 \cdot 8 \cdot 3
 \end{array}$$

Suppose a piece of timber 45 feet long, 1 foot 9 inches deep, and 1 foot 6 inches thick, how many cube feet does it contain? Multiply by 9 and by 5, because 5 times 9 is 45, and the content is 118 feet, 1 inch, 6 parts.

$$\begin{array}{r}
 1 \cdot 9 \\
 1 \cdot 6 \\
 \hline
 0 \cdot 10 \cdot 0 \\
 1 \cdot 9 \cdot 0 \\
 2 \cdot 0 \cdot 7 \cdot 0 \\
 \hline
 23 \cdot 7 \cdot 6 \\
 \hline
 118 \cdot 1 \cdot 6
 \end{array}$$

Suppose a piece of timber 36 feet long, 2 feet 9 inches deep, and 1 foot 7 inches thick, how many cube feet are contained in it? Multiply by 6 and 6, because 6 times 6 is 36, and so for any other. The content is 156 feet 9 inches.

$$\begin{array}{r}
 2 \cdot 9 \\
 1 \cdot 7 \\
 \hline
 1 \cdot 10 \cdot 7 \cdot 3 \\
 2 \cdot 9 \\
 \hline
 4 \cdot 2 \cdot 4 \cdot 3 \\
 \hline
 46 \cdot 1 \cdot 6 \\
 \hline
 156 \cdot 9
 \end{array}$$

Suppose a piece of timber 32 inches one way and 20 inches the other way, at the great end, 10 inches one way and 6 inches the other way, at the little end, and 18 feet long, how many cube feet are contained in the piece? To find the solid content, multiply the two bases together, and out of that product extract the square root, and to that root add the area of the two bases, and that sum multiplied by one third of the height, will give the solid content of any frustum, square or rectangular.

10 32
6 20
60 640 area of the great base.
60 area of the lesser base.

38400
1
195.959 foot.
640. area of great base,
60. area of lesser base.
895.959 sum, or mean area
6 one third of the
[height.

12) 5375.754
3902) 37500
35181
12) 4479.79
39185) 231900
195925
391909) 3597500
3527181
70319

*A Table, shewing how much in Length will make a Cube-Foot
of any common Scantling, cut fit for building.*

Scantling.			Number of feet in length to one cube foot.	
Inches.		Inches.	Feet.	Inches.
2	by	2	36	0
4	—	3	12	0
4	—	4	9	0
6	—	4	6	0
8	—	6	3	0
9	—	6	2	8
10	—	6	2	4
12	—	9	6	0
12	—	9	1	4
18	—	16	6	0
18	Square	18	6	5
24	ditto	24	16	0
36	ditto	36	6	0
48	ditto	48	144	0

Or, the area of the end must be a divisor to 1728,
which will give the length of a cube foot, Men-

Measurement of Board, Plank, &c.

Suppose a board to be 18 feet 6 inches long, and 9 inches wide. Multiply 9 inches into the length, and divide the product by 12, the answer will be 13 feet, 10 inches, 4 parts, 6 seconds. To work this, say, 6 times 9 is 54, 6 and carry 4, then 9 times 8 is 72 and 4 I carry is 76, set down 6 and carry 7, saying, 9 times 1 is 9 and 7 I carry is 16, which is 166.6, to be divided by 12. The answer is 13 feet 10 inches.

$$\begin{array}{r}
 18.6 \\
 \times 9 \\
 \hline
 12)166.6 \\
 \hline
 13.10.6
 \end{array}$$

Suppose a board 14 feet long and 10 inches 6 parts wide. In this work take the aliquot parts of 12 in 14 feet, and the answer will be 12 feet 3 inches.

$$\begin{array}{r}
 14.0 \\
 \times 10.6 \\
 \hline
 7.7 \\
 4.8 \\
 \hline
 12.3
 \end{array}$$

Suppose a board 24 feet 6 inches long and 1 foot 6 inches wide. The content is 36 feet 9 inches.

$$\begin{array}{r}
 24.6 \\
 \times 1.6 \\
 \hline
 24.6 \\
 12.3 \\
 \hline
 36.9
 \end{array}$$

Suppose a board 30 feet 9 inches long and 1 foot 9 inches wide. To work this, begin with 1 foot, in the bottom line, saying, once 9 is 9, once 0 is 0, and once 3 is 3. Then for the 9 inches take half the 30, which is 15, and the half of 9 is 4 and 6 parts; then there is 3 inches left in 9, which is one fourth of 12; then say, the fours in 30 are 7 and 2 over, which is 24 inches; carried to 9 makes 36 inches, then the fours in 36 are 9; add all up, the sum is 53 feet, 9 inches, 9 parts.

$$\begin{array}{r}
 30.9 \\
 \times 1.9 \\
 \hline
 30.9 \\
 15.4.6 \\
 7.8.3 \\
 \hline
 53.9.9
 \end{array}$$

Mensuration of Plank, superficial and solid.

Suppose a plank 15 feet long, 10 inches wide, and an inch and half thick, how much superficial on the face, and how much solid?

Answer. 15 feet, 6 inches, 9 parts, solid.

Suppose a plank 21 feet long, 11 inches wide, and 2 inches thick, the superficial content is 19 feet 3 inches, and the solid or cube 3 feet, 2 inches, and 6 parts. To work this, multiply 21 by 11, and the product is 231 long inches, which divide by 12 and it gives 19 feet 3 inches. For the 2 inches thick, take the sixes in 19 feet 3 inches, which gives 3 feet, 2 inches, and 6 parts, cube.

Suppose a plank 18 feet long, 1 foot 4 inches wide, and 2 inches and a half thick, how many feet superficial on the face, and how many feet cube in the plank? For the 2 inches take the sixth part of 24, which is 4; then for the 6 parts take the 24th part of 24, which is 1; add them together and they make 5 feet cube.

Or, if the area of the end be multiplied into the length, 18 feet, it will give 5 feet cube.

Suppose

Suppose a plank 25 feet long, 1 foot 8 inches wide, and 3 inches thick, how much superficial on the face, and how many feet cube? To work this, say, once 25 under 1 foot, then for the 8 inches, take the two-thirds of 25, which is 16 feet 8 inches, add them together, and they make 41 feet 8 inches, the superficial content. Then, for the 3 inches thick, take one fourth part of 41 feet 8, which is 10 feet 5 inches, the cube.

$$\begin{array}{r}
 25 \text{ length.} \\
 1 \text{ foot } 8 \text{ width} \\
 \hline
 25 \\
 16 \cdot 8 \\
 \hline
 41 \cdot 8 \text{ super.} \\
 3 \text{ thick.} \\
 \hline
 10 \cdot 5 \text{ cube ft}
 \end{array}$$

Suppose a plank 32 feet long, 1 foot 9 inches wide, and three inches and a half thick, how many feet superficial on the face, and how many solid or cube? As 1 will not multiply, set down 32, then for the 9 inches take a half and a fourth of 32, set them down, and add all up, the sum is 56. Then for the thickness, 3 inches, take a fourth part of 56, which is 14, and for the 6 parts take the twelves in 28, which is the half of 56, and that will be 2 feet 4 inches, add all up, the cube content is 16 feet 4, or 1 third of a foot, equal to 576 cube inches.

$$\begin{array}{r}
 32 \text{ length} \\
 1 \cdot 9 \text{ width} \\
 \hline
 32 \\
 16 \\
 8 \\
 \hline
 56 \text{ superficial} \\
 3 \cdot 6 \text{ thickness} \\
 \hline
 14 \\
 2 \cdot 4 \\
 \hline
 16 \cdot 4 \text{ cube}
 \end{array}$$

Suppose a plank 36 feet long, 1 foot 10 inches wide, and 4 inches thick, how many feet superficial on the face, and how many feet cube? For the 1 foot set down 36, and for the 10 inches the half and a third, which is 18 and 12, add all up, the sum is 66 feet superficial. For the 4 inches thick take a third of 66, which is 22, the cube feet in the plank.

$$\begin{array}{r}
 36 \cdot 0 \\
 1 \cdot 10 \\
 \hline
 36 \\
 18 \\
 12 \\
 \hline
 66 \text{ superficial} \\
 4 \text{ thickness} \\
 \hline
 22 \cdot 0 \text{ cube}
 \end{array}$$

Suppose

Suppose a board or plank to be 25 feet long and 36 inches wide, at one end, and 24 inches wide at the other end; the superficial content is required. Multiply the two ends together, and out of that product extract the square root, which is a mean breadth to be multiplied by the length for the superficial content.

$$\begin{array}{r}
 36 \\
 24 \\
 \hline
 144 \\
 72 \\
 \hline
 864
 \end{array}
 \begin{array}{r}
 29.39 \text{ mean breadth} \\
 25 \\
 \hline
 146.95 \\
 587.8 \\
 \hline
 [long inches]
 \end{array}$$

864 (29 39 root. 12) 734.75 superficial content in

$$\begin{array}{r}
 4 \\
 \hline
 49)464 \\
 441 \\
 \hline
 2300 \\
 1749 \\
 \hline
 5869)55100 \\
 52831 \\
 \hline
 2269
 \end{array}$$

61.229 superficial feet

Number of superficial Feet to a Load of Plank, at different Thicknesses.

Thickness. Inches.	Number of superficial Feet to a Load.
1	600
1½	400
2	300
3	200
4	150

To find how much in length will make a superficial foot of board or plank, at any width, the width of the board, in inches, must be a divisor to 144, the inches in a superficial foot, and the quotient will be the length in inches. For example.

$$\begin{array}{r}
 18)144 \\
 144 \\
 \hline
 \end{array}$$

(8 inches long make 1 foot superficial, and so for [any other.]

$$\begin{array}{r}
 16)144 \\
 144 \\
 \hline
 \end{array}$$

(9 inches long make 1 foot superficial.

*The Mensuration of Carpenters-Work, such as Centering
Floors, Roofs, Partitions, &c.*

Suppose a cellar to be groined in the length 18 feet 6 inches, and the girt 15 feet 9 inches, how many squares of centering?

Note, in measuring groins, girt the angles or gathering, and for every foot in girt count 1 superficial foot.

The content is 2 square, 91 feet 4 inches, and 6 parts.

Suppose a floor to be 24 feet 10 inches, by 18 feet 8 inches, and the content 4 square, 63 feet, 6 inches, 8 parts. To work this, multiply 18 by 24; then for the 8 inches take two thirds of 24 feet 10 inches, which is 16 feet, 6 inches, and 8 parts; then, for the 10 inches take half and a third of 18, which is 9 and 6; add all up together, the sum is 463 feet 6 inches, and 8 parts, or 4 squares, 63 feet, 6 inches, and 8 parts.

Note, The labour is in general done by the square, and sometimes floors, partitions, roofs, &c. are done by the foot cube, for labour only.

Guttering is done by the foot superficial, at so much per foot, gutter and bearers.

Suppose a gutter 75 feet 6 inches long and 3 feet 6 inches wide at one end, and 10 inches wide at the other end; the mean breadth will be 2 feet 2 inches, then adding 6 inches for the lear-board, it will be 2 feet 8 inches. *Note,* the drop or fall $\frac{1}{4}$ of an inch to a foot.

Multiply 2 by 75, and for the 8 inches take two thirds of 75, which is 50, and two thirds of 6 inches is 4 inches; then for the 6 inches, take the half of 2, which is 1; add all up; the sum is 201 feet 4 inches.

Note, All sorts of boarded floors are measured by the square. Stairs are measured by the foot superficial, girting over the steps and nosings for the first dimension, and the length of the step for the other dimension. The rail is to be taken at per foot run, and the ramps, twist, &c. to be allowed three times the strait. If they are very small and close, four times will but just do. Ban- nisters at per piece. Brackets at per piece, according to the work there is in them. Architrave string-boards at per foot superficial moulded work.

Wainscot is done by the yard square. Take the girt of the room round for one dimension, and the height for the other dimension, and multiply them together, the product will give the content in feet and inches. Divide that sum by 9, the square feet in a square yard, and the content will be yards and feet. For the doors, windows, chimnies, &c. their dimensions must be taken and squared, and then deducted from the content of the room, and the remainder will be the content in yards and feet for the wainscoting, or any other finishing the room may be done with.

Suppose the girt of the room be 84 feet 10 inches, the height 14 feet 6 inches, and the deduction for doors, windows, &c. to be 18 yards 8 feet, the true content of wainscot, or finish- ing, will be 117 yards 7 feet.

84 . 10

14 . 6

336

84

42 . 5

7

4 . 8

9)1230 . 1

136 . 6 from

18 . 8 take

117 . 7 cont.

Note, In taking the dimensions of all kinds of mould- ing, as bases, subbases, architrave, door-caps, chimney- caps, bases and caps of columns, imposts, &c. they are also to be girt over the faces of the mouldings with a line, squeezing the line into all the quirks and sinkings; and that multiplied by the length, or other dimensions, will give the superficial content in feet and inches.

Suppose

Suppose the girt over the face to be 7 inches and a half, and the length 45 feet 9 inches. To work this, take, for the 7 inches, a third of 45 feet 9 inches, which is 15 feet 3 inches, then there is a fourth left in 7; take the fourth of 45 feet 9 inches, which is 11 feet, 5 inches, 3 parts, and for the 6 parts take the half of 45 feet 9 inches, which is 22 inches, 10 parts and 6 seconds, then take the twelves in that, which is 1 foot, 10 inches, 10 parts, and 6 seconds, add all up, the sum is 28 feet, 7 inches, 1 part, 6 seconds.

$$\begin{array}{r}
 45 \text{ feet } 9 \text{ inches} \\
 \underline{15 \text{ feet } 3 \text{ inches}} \\
 30 \text{ feet } 6 \text{ inches} \\
 \underline{11 \text{ feet } 5 \text{ inches } 3 \text{ parts}} \\
 19 \text{ feet } 1 \text{ inch } 3 \text{ parts } 6 \text{ seconds} \\
 \underline{22 \text{ inches } 10 \text{ parts } 6 \text{ seconds}} \\
 28 \text{ feet } 7 \text{ inches } 1 \text{ part } 6 \text{ seconds}
 \end{array}$$

Note, In measuring floors, the well-holes for stairs, chimnies, &c. must be deducted out of the whole: Likewise doors, windows, &c. in partitions, except, by agreement, they are to be included.

In measuring roofs, there should not be any deductions for chimnies, sky-lights, &c. for they give more trouble to make good to them than the stuff is worth to cover them.

MENSURATION OF BRICK-WORK.

The particulars to be observed in measuring brick work are, that it is by the rod of 16 feet and a half square, whose content is 272½ in one Rod of work, at the standard thickness of one brick and a half; and, if the walls are more or less in thickness, they must be reduced to brick and a half by multiplying the number of half bricks in the thickness of the wall into the superficial content of the wall, and dividing that product by 3, the number of half bricks in the standard thickness, and the quotient will be the content in reduced brick work.

Note, When the thickness of the wall is one brick and half, it is ready reduced. Always observe the several

several thickneses of walls, and that you make every deduction out of the proper thicknes. When two walls are measured that form an angle, the length of one is to be taken to the inside of the wall, and the other to the outside as *fig. 6, plate I.*

Of measuring Chimnies.

Chimnies are measured in the same manner as walls; the vacancy between the jambs one way, and the floor and mantle the other way, must be deducted. The funnels or shafts are measured as solid, on account of the trouble and the pargeting withinside, &c.

In measuring the trimmers that support the hearths, take the length and girt of the arch, and count them half a brick thick.

Suppose a front wall to be 24 feet 9 inches in length, 14 feet 6 inches in height, and two bricks thick, as *fig. a plate 1.* and a part of the said wall with a return wall 18 feet 6 inches long, by the same height, how many rods of brick work?

The content 3 rods, 3 quarters, 9 feet, reduced brick work. Suppose there to be 3 windows in the front wall, on the outside, 7 feet by 3 feet 6, in half a brick thick; the inside 7 feet 4 by 4 feet 2, in a brick and a half thick. These dimensions must be squared, and the contents be deducted from the whole content of the first dimensions, and the remainder will be the true content of the wall.

Sub-note of measure Walls of Chimney-Shafts as solid.

Suppose the shaft to be 6 feet by 3 feet, and 12 feet high. Multiply the length by the thicknes, and that product by the height; then multiply that product by 8, and divide by 9, and it gives the content in feet reduced to standard thicknes.

$$\begin{array}{r}
 53 \cdot 3 \\
 14 \cdot 6 \\
 \hline
 772 \cdot 1 \cdot 6 \\
 4
 \end{array}$$

$$\begin{array}{r}
 373088 \cdot 6 \cdot 6 \\
 2731029 \cdot 6(3) \\
 816 \\
 \hline
 68213(8) \\
 204
 \end{array}$$

The method for measuring angular chimnies is something different from square chimnies, of which I shall give an example. *Fig. e* is a chimney standing in the angle of the room and equal each way. In this case, take half the width of the breast *a b*, suppose that to be 3 feet, and the height of the story 10 feet 3 inches, then the dimensions will stand thus

$$\begin{array}{r} 10 \text{ feet } 3 \text{ inches} \\ 3 \\ \hline 30 \text{ feet } 9 \text{ inches} \end{array}$$

superficial content.
8 half bricks.

then consider how many half bricks are contained in the said half breast. There are 8 half bricks, or 4 bricks in the thickness of the wall, which multiply by the superficial content of the breast, and divide by 3, and it gives 82 feet reduced work.

Note, If the angle be not square, the usual way is, to lay out the angle parallel to the walls, as *c* and *d*, and take one of those whole lines for the width of the breast and half the other for the thickness, then the whole line multiplied by the height of the story gives the superficial content. Then multiply the number of half bricks in the half-line into the superficial content, and divide that product by 3, it gives the content in reduced brick-work. Sometimes there is a bur for the under breast, as in *fig. d*, and that is measured in the respective thickness as a wall.

Of Measuring square Chimnies, as Fig. b, Plate I. standing without a Party-Wall being joined to them.

Multiply the width of the breast by the thickness, and that product by the height of the story; then multiply the last product by 8 and divide by 9, the quotient will be feet reduced.

Fig. a is the section of a wall with the height of every story figured, and the thickness of the wall in every story

story, from the bottom footing to the parapet. The length is supposed to be 45 feet, the thickness of the parapet $1\frac{1}{2}$ brick, the attic 2 bricks, the one pair of stairs $2\frac{1}{2}$ bricks, the principal 3 bricks, the basement $3\frac{1}{2}$ bricks, the foot-lace $\frac{1}{2}$ of a brick, the plinth $\frac{1}{4}$ ditto, and the footing 5 bricks thick.

A Sketch of the Dimension-Book for Brick-work.

Rule 4 columns, as A B C D; and, when you take the dimensions, place them as follow: the dimensions given in the column B, the number of bricks the wall is thick in the column C, and the content when squared in the column D. The figures in the column A denote the numbers of each dimension, when more than one.

A.	B.	C.	D.	
		Bricks.		
3	45 . 8 2	5	274 . 0	Footing in foundation.
2	45 . 4 10	3 $\frac{1}{2}$	906 . 8	Basement-story
2	45 . 8 13	3	1212 . 0	Principal story.
	45 . 0 13	2 $\frac{1}{2}$	585 . 0	One pair of stairs.
	45 . 0 10	2	450 . 0	Attic.
	45 . 0 4	1 $\frac{1}{2}$	180 . 0	Parapet.
	45 . 4 7	$\frac{1}{2}$	317 . 0	Foot-lace.
	45 . 4 3	$\frac{1}{4}$	136 . 0	Plinth to basement.
4	45 . 4 3	$\frac{1}{8}$	90 . 8	String, or fasci in front.
4	4 . 2 3	1	58 . 4	Deduct window, outside base-

A.	B.	C.	D.	
		Bricks.		
4	6 . 6	$\frac{1}{2}$	91	o Deduct window outside prin-
	3 . 6			
4	6 . 0	$\frac{1}{2}$	84	o Deduct windows outside, one
	3 . 6			
4	4 . 0	$\frac{1}{2}$	56	o Deduct windows in the attic
	3 . 6			
4	4 . 4	$2\frac{1}{2}$	72	o Deduct basement windows in-
	4 . 2			
4	6 . 10	$2\frac{1}{2}$	113	o 10 Deduct windows inside to
	4 . 2			
4	6 . 4	2	105	o 4 Deduct windows inside, one
	4 . 2			
4	4 . 4	$1\frac{1}{2}$	72	o 2 Deduct windows inside to the
	4 . 2			

46 feet run of played foot-lace. 76 feet superficial gauge arches set in putty. 160 feet run of rubbed returns.

Bill of Materials

24 Rods & quarters reduced brick work, at per rod — 84

76 Feet of superficial gauge arches, at per foot superficial — 204

160 Feet run of rubbed returns, at per foot — 44

92 Feet run of played foot-lace, at per foot — 344

200 3 10

For the Prices in general see the Builder's Golden Rule, and the Estimate annexed to it.

Abstract of Brick-Work.

One Brick & $\frac{1}{2}$ Column. Add.	Deduct.	One Brick Col. Add.	Deduct.	Superficial Gauge Ar- ches.
822 0	72 0	137 0	58 4	76
1813 4	113 10	456 4	45 6	
2430 0	105 0	585 0	42 0	
585 0	72 2	225 0	28 0	
450 10		79 1	72 0	
180 0	363 0	34 0	113 6	Run of
		15 0	52 6	splay foot-
fr. 6280 4		fr. 4531 5	411 4	lace.
take 363 0		take 411 4		138
5917 4		1120 1 1		
		2		
		3) 2240		Rubbed
		746 2		returns.
		5917 4 r. 9.		160
		272) 5663 6(24 2		
		544		
		1223		
		1088		
		68) 135		

Bill of Bricklayers-work.

	£.	s.	d.
24 Rods 2 quarters reduced brick work,			
at per rod — — — 8l.	196	0	0
76 Feet of superficial gauge arches, at			
per foot superficial — 20d.	6	6	8
160 Feet run of rubbed returns, at			
per foot — — — 4d.	2	13	4
92 Feet run of splayed foot-lace, at			
per foot — — — 3½d.	1	3	10
	206	3	10

For the Prices in general see the Builder's Golden Rule, and the Estimate annexed to it.

P I

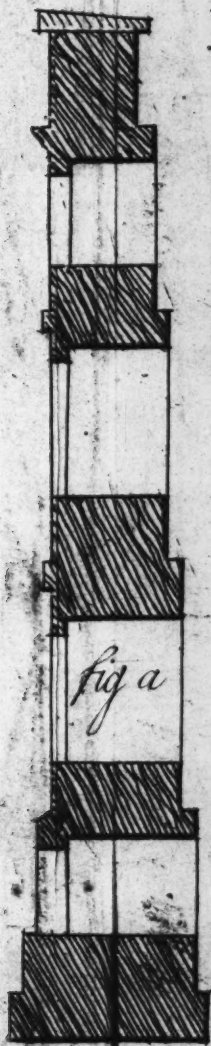


fig a

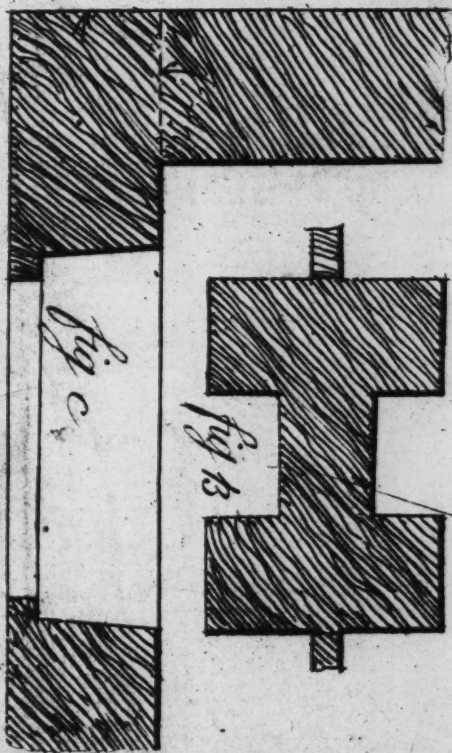
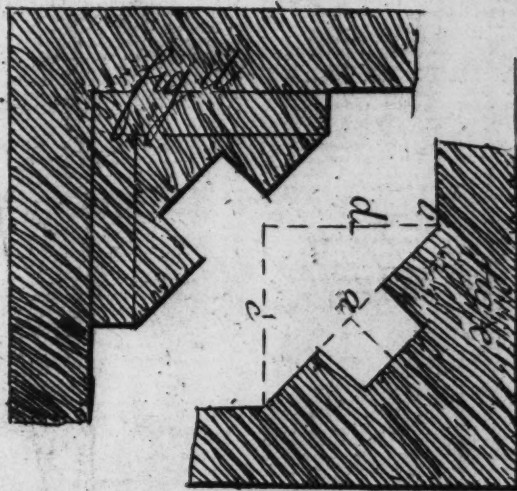


fig c



fig d

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To reduce the different Thicknesses of Brick Walls to the Standard of one Brick and Half or one Brick thick, as in the Abstract, proceed as follows:

Note, The first dimensions, 274,0, must be trebled in the abstract.

The wall is 5 bricks thick, which is 3 standards thick and one half brick, which is 822 feet, in a brick and a half thick, as placed in the abstract. Take half 274, which is 137 feet in one brick thick, and place this in the one brick column, as in the abstract, and so in any thickness. If a wall is two standards and $\frac{1}{2}$ brick thick, double the superficial content, and place it in the one brick and half column; for the half brick, halve the superficial content, and place the sum in the one brick column. So, for half a brick above the standard, place half the superficial content to the abstract; if one brick above the standard, place the whole superficial to the abstract. If a quarter of a brick thick, place a quarter of the superficial content to the abstract. If a sixth of a brick thick, place one sixth of the superficial content to the abstract, &c. By this method, you reduce all the small thicknesses to one brick thick. When that is done, add the one brick column up; multiply the sum by 2, divide the product by 3, and the quotient will be the number of feet reduced. Proceed in the same manner to reduce the deductions to one brick and a half and one brick thick. See the abstract.

The Dimension-Book for Carpenters Work.

Note, the first column A, contains the number of times when more than once, the column B the dimensions given, and the column C the contents when squared.

A.	B.	C.	
8	11 6	32	Rough fir framed in roofs, floors, &c.
	8 1		
	6		
4	7 6	20 8	King-post.
	6		
	4		
4	14 0	9 4	Struts to the roof.
	6		
	4		
4	18 0	35 0	Tie-beams to the roof.
	10		
	7		
64	11 6	74 4	Small rafters.
	5		
	3		
	60 0	20 0	Purlines to the roof.
	8		
	6		
6	24 0	20 8	Binding-joist to the cieling floor.
	7		
	4		
60	3 0	9 4 1/2	Ceiling-joist.
	3		
	2 1/2		
12	18 0	27 0	Girders in the chamber floor.
	12		
	9		
18	24 0	120 0	Binding-joist to ditto.
	8		
	5		

A.	B.	C.			
32	16 0	53 4	Bridging-joist to the chamber floor.		
	3				
60	3 0	9 4½	Ceiling-joist to ditto.		
	3				
	2½				
	18 9	6 8	Bottom-plate to the partitions in the		
	6		[chambers]		
	18 0	5 7	Top-plate to ditto.		
	9				
	6				
	18 0	5 9	Principal post to ditto.		
	9				
	5				
3	11 6	11 8	Quarters to ditto.		
	6				
	4				
10	11 6	2 8	Quarters to partitions.		
	5				
	3				
2	13 0	2 8½	Beams to partitions.		
	5				
	3				
2	52 0	21 4	Raising-plates.		
	2				
	6				
32	0	14 2½	Pole-plates.		
	7				
	5				
150	0		Run of bend in the walls.		
	4				
	2½				
2	18 0	30 0	Girders in the principal floor.		
	12				
	10				

A.	B.	C.							
18	24 0 8 5	120 0		Binding-joint to the principal floor.					
32	16 0 5 3	53 0		Bridging-joint.					
60	3 0 3 2½	9 4½		Ceiling-joint.					
2	6 0 10 4	3 4		Trimmers to the hearths.					
	12 10 10 6	134 9		Yellow whole-deal clean doweled floors.					
	18 10 15 6	291 11		Ditto.					
	15 6 12 5	192 5		Second-best yellow, nailed, one edge in sight.					
	12 3 10 4	126 7		Inch white-deal folding floors.					
	65 0			Foot run of narrow ground under-mouldings					
	18 0 4	12 0		Superficial inch grounds framed to doors.					
	24 0 3	8 0		Superficial ditto to windows.					
	5 3 9	3 9		Grounds to chimnies.					
	75 0 2 3	158 9		Inch-deal dado keyed.					
	75 0 4	25 0		Surbase moulding quirked.					
	75 0 2½	15 7½		Ditto base mouldings.					

A.	B.	D.	
48 8 8	32 0	Door architraves.	
48 0 6	24 0	Window ditto.	
3 7 7 3 6	79 7½	Two inch deal doors, 6 pannels, ovolo and flat on both sides, and moulding on the pannels.	
6 7 0 1 2	49 0	Inch and half deal shutters, quirkogee and bead, small moulding on the pannels, bead &c but on the back.	
3 7 0 3 5	71 9	¾ Wainscot sash, astragal and hollow, with oak fills and wainscot pully-pieces.	
2 5 9 9	8 7½	Quirk mouldings to chimney-caps.	
2 4 10 1 10	17 8	Ditto to door-caps.	
70 0 5	39 1	½ Plinth.	
3 0 4 0 1 2	14 0	Soffets to windows.	
6 7 0 1 1	45 6	Back window lining, inch deal, framed, bead and but.	

Abstract of Carpenters Work.

Rough fir framed in floors, roofs, &c.	Whole- deal clean dowell- ed floor.	Framed grounds under mould- ings, su- perficial, to doors, windows, &c.	Inch deal dado, key- ed, and joints backed.	Two inch deal doors, ovolo flat pannels on both sides, and small astragals on the pannels.
32 0	134 9			
26 8	291 21		158 9	
9 4	426 8			79 7½
35 0				
74 4		12 0	Superfici- al quirk	
20 0		8 0	mould- ings to	Inch & ½ deal
28 0	Ditto	3 9	base, fur- base, ar- chitraves, &c.	shutters,
9 4½	second	23 9		quirk, ogée and
27 0	bead			bead, small
120 0	nailed			moulding on
53 4	one			the pannels,
9 4½	edge.	Run of		bead and but
6 8		narrow		on the back.
5 7	192 3	ground	25 0	
5 9		under	15 7½	49 3
11 8		mould- ings, 2½	32 0	
2 8½	Inch	inches	24 0	
21 4	white	wide.	96 7½	Soffers to ditto.
14 2	deal			14 0
30 0	folded			2 1
120 0	floor.	65 0		
53 0				
9 4½	186 7	Superfici- al ½ deal plinth.	Superfici- al quirk mould- ings to chimney- caps, door- caps, &c.	Ditto back lining, inch deal, bead & but.
3 4		29 1	8 7½	46 9
722 0			17 8	
			26 3½	Two inch fash and frame, as- tragals and hol- low oak fills, and wainscot pully-stiles.
Run of bond in walls.				71 9
150 feet.				
4				
2½				

For the Prices in general see the Builder's Golden Rule.

Bill

Bill of Carpenters Work.

			<i>l.</i>	<i>s.</i>	<i>d.</i>
150	Foot run of bond in walls 4 by 2½ at per foot	— — —	2½	1	5 00
722	Feet of cube fir, framed in floors, Roofs, &c. at per foot	— — —	2½	8½	96 5 14
4	Square 26 feet 8 inches clean dowelled floor, at per square	— — —	5½	15	24 10 3
1	Square 92 feet second-best floor nailed one edge, at per square	— — —	3½	15	7 4 0
1	Square 26 feet inch white deal folded flooring, at per square	— — —	1½	12½	2 2 3
65	Feet run of grounds under mouldings, 2½ inches wide, at per foot	— — —	2½	0	13 6½
23	Feet 9 inches superficial, inch deal framed grounds to doors, windows, &c. at per ft	— — —	7d.	0	13 9
159	Feet superficial deal dado, keyed, and joints backed, at per foot	— — —	9d.	5	19 3
96	Feet 7½ inches superficial quirked mouldings, at per foot	— — —	1½	2d.	4 3 6
26	Feet 3 inches superficial moulding to chimney-caps, doors, &c. at per ft	1s. 6d.	1	19	4½
79½	Feet superficial 2 inch deal doors, ovolo flat on both sides, and astragals on the pannels, at per foot	— — —	1s. 4d.	5	6 6
49	Feet superficial inch and half deal shutters, quirkogee and bead, small moulding on the pannels, bead & but on the back, at per ft	1s. 6d.	3	13	6
14	Feet superficial soffet to ditto, at per ft	9d.	0	10	6
29	Feet superficial ¾ deal plinth, at	5d.	0	12	1
45	Feet 6-inch deal framed in back linings to windows, at per foot-superficial	8d.	1	10	0

156 8 10

Multiplication

MULTIPLICATION OF FEET AND INCHES.

Question 1.

Suppose a door
to be 8 feet 6 inches
9 parts high by 4 ft
3 in. 4 ptswide, how
superficial feet are
contained therein?
The content in feet
and inches is 36 ft
7 in. 6 pts & 6 sec.
and by decimals
36 ft 7 in. $\frac{1}{10}$ of an
inch.

The same by decimals.

8 6 9	8.565
4 3 4	4.270
34 3 0	59.9550
2 1 8 3	17.150
2 10 3 0	34.300
36 7 6 6 0	36.912550

Question 2.

Suppose a floor
to be 18 feet 10 in-
ches by 14 feet 6
inches, the con-
tent is 2 square 73
feet.

The same by decimals.

18 10	18.83
14 6	14.50
72	94.150
18	7532
9 5	18.83
7 0	
4 8	2)73.0350
2)73 1	

Question 3.

Suppose a floor
to be 36 feet 9 in-
ches by 18 feet 4
inches, the content
is 6 squares 73 feet
9 inches.

The same by decimals.

36 9	36.75
18 4	18.33
288	11025
36	11025
12 3	29400
9	3675
4 6	
6)73 9	6)73.6275

To work the first question, multiply 4 feet by 9 parts, saying 4 times 9 is 36, 0 and carry 3, then 4 times 6 is 24 and 3 I carry is 27, set down 3 and carry 2, saying 4 times 8 is 32 and 2 I carry is 34; then begin with 3, saying 3 times 9 is 27, set down 3 and carry 2, saying 3 times

times 6 is 18 and 2 I carry is 20, set down 8 and carry 1, saying 3 times 8 is 24 and 1 I carry is 25, which is 2 feet 1 inch to be set down; then multiply 4 parts by 9, saying 4 times 9 is 36, set down 0 and carry 3, saying 4 times 6 is 24 and 3 I carry is 27, set down 3 and carry 2, saying 4 times 8 is 32 and 2 I carry is 34, which is 2 inches and 10 parts; add all up together, the content is 36 feet, 7 inches, 6 parts, 6 seconds.

To work the same by decimals is the same as multiplication of whole numbers, only pointing off as many decimals in the product as there are in the multiplicand and multiplier. To find the value of the fraction cut off from the product, multiply by 12, and point off as many places to the right-hand as you had decimals, and what remains to the left is inches; then multiply the decimal left by 4, and point off as before, and what is to the left-hand, if any, are quarters.

To work the second question, multiply feet by feet, then take the aliquot parts of twelve; that is, as six is the half of twelve take 6 in the top-line, saying the half of 18 is 9, and the half of 10 is 5; then I take 10 inches in 14 feet; in 10 inches there is a half and a third of 12; then say, the half of 14 is 7, and the third of 14 is 4 and 2 over, which is two thirds of 12, or 8 inches; add all up, and the sum is 273 feet 1 inch, the content, the same as by decimals; and so for any other, in the following.

Suppose a floor or passage to be 36 feet 5 inches 4 parts by five feet 4 inches 6 parts. To work this question, multiply 5 feet into the top-line, beginning with 4 parts, saying 5 times 4 is 20, set down 8 and carry 1, saying 5 times 5 is 25 and 1 I carry is 26, set down 2 and carry 2, saying 5 times 6 is 30 and 2 I carry is 32, set down 2 and carry 3, saying 5 times 3 is 15 and 3 I carry is 18, to be set down; then for the 4 inches, I take a third part of the top-line, saying, a third of 36 is 12, and the third of 5 is 1, and two twelfths over, which is 24, and the 4 parts make 28; the third of that is 9 and one third, which is 2, then for the 6 parts I

I

take

furing the same are as follow; to make deductions on the sides or party for doors, windows, chimnies, and girders that lie below the cielings, to be deducted if the workman find materials, otherways not. For rendering between quartering, if the workman find materials, one fifth part must be deducted for the quarters, but if labour only, you must measure the whole, for the workman would do it much sooner, if no quartering had been there.

Note, all mouldings in plaister-work are done by the foot superficial, and measured the same as the carpenters, viz. girt over the face with a line.

P A I N T E R S

Measure their work by the yard square, the same as plaisterers, only with this difference, they measure all edges where the brush goes to carry colours.

G L A Z I E R S

Measure their work by the superficial foot; these dimensions are taken in feet and hundredth parts of a foot, according to the rules of decimals; all oval and circular windows they measure as square, to allow for waste of time, &c.

A Collection of sundry Things necessary to be known by all concerned in building.

A load of earth is one solid yard, equal to 27 cube feet.

A hundred weight of iron nails, lead, &c. is 112 lb.

A hundred of deals or nails, six score, or 120.

A bundle of 5 feet laths, five score; of four feet, six score.

A fother of lead is 19 hundred and a half, or 2184 pounds, avoirdupoise weight.

A bale of paper is ten reams, a perfect ream 20 quires, or 500 sheets; one perfect quire 25 sheets.

The Weight of a Cube Foot of sundries.

	Pounds.	Ounces.
A cube foot of lead weighs	707	0
Ditto of copper	562	4
Ditto of brass	521	8
Ditto of cast brass	500	0
Ditto of steel	490	7
Ditto of iron	477	5
Ditto of glass	161	2
Ditto of marble	196	3
Ditto of Portland stone	140	0
Ditto of clay moderately moist	112	0
Ditto of gravel of common moisture	96	0
Ditto of sea-water	64	1
Ditto of river-water	62	3
Ditto of dry oak	37	8

Of a circular Foot.

A circular foot contains 113 square inches and one seventh of an inch, that is, a circle that is 1 foot diameter: in a cylindrical foot are 1573 cube inches and $\frac{1}{7}$ of an inch, that is, 12 times 113, one seventh of the square inches in a circular foot.

A cylindrical foot of sea-water weighs 50 $\frac{1}{2}$ pounds.

Ditto of fresh water 49 pounds and $\frac{1}{10}$.

A cube foot contains 6 gallons and a pint of liquor.

A circular foot contains 5 gallons and a half and $\frac{3}{4}$ of a pint.

Weight of Square
Iron Bars, 1 foot
in length.

Weight of Flat Iron bars,
one foot in length.

Square.	lb.	q.	oz.	width. Inches.	thick- ness.	lb.	q.	oz.
1	0	0	0	1	1	1	0	0
2	0	0	0	1	2	2	0	0
3	0	0	0	1	3	3	0	0
4	0	0	0	1	4	4	0	0
5	0	0	0	1	5	5	0	0
6	0	0	0	1	6	6	0	0
7	0	0	0	1	7	7	0	0
8	0	0	0	1	8	8	0	0
9	0	0	0	1	9	9	0	0
10	0	0	0	1	10	10	0	0
11	0	0	0	1	11	11	0	0
12	0	0	0	1	12	12	0	0
13	0	0	0	1	13	13	0	0
14	0	0	0	1	14	14	0	0
15	0	0	0	1	15	15	0	0
16	0	0	0	1	16	16	0	0
17	0	0	0	1	17	17	0	0
18	0	0	0	1	18	18	0	0
19	0	0	0	1	19	19	0	0
20	0	0	0	1	20	20	0	0
21	0	0	0	1	21	21	0	0
22	0	0	0	1	22	22	0	0
23	0	0	0	1	23	23	0	0
24	0	0	0	1	24	24	0	0
25	0	0	0	1	25	25	0	0
26	0	0	0	1	26	26	0	0
27	0	0	0	1	27	27	0	0
28	0	0	0	1	28	28	0	0
29	0	0	0	1	29	29	0	0
30	0	0	0	1	30	30	0	0
31	0	0	0	1	31	31	0	0
32	0	0	0	1	32	32	0	0
33	0	0	0	1	33	33	0	0
34	0	0	0	1	34	34	0	0
35	0	0	0	1	35	35	0	0
36	0	0	0	1	36	36	0	0
37	0	0	0	1	37	37	0	0
38	0	0	0	1	38	38	0	0
39	0	0	0	1	39	39	0	0
40	0	0	0	1	40	40	0	0
41	0	0	0	1	41	41	0	0
42	0	0	0	1	42	42	0	0
43	0	0	0	1	43	43	0	0
44	0	0	0	1	44	44	0	0
45	0	0	0	1	45	45	0	0
46	0	0	0	1	46	46	0	0
47	0	0	0	1	47	47	0	0
48	0	0	0	1	48	48	0	0
49	0	0	0	1	49	49	0	0
50	0	0	0	1	50	50	0	0
51	0	0	0	1	51	51	0	0
52	0	0	0	1	52	52	0	0
53	0	0	0	1	53	53	0	0
54	0	0	0	1	54	54	0	0
55	0	0	0	1	55	55	0	0
56	0	0	0	1	56	56	0	0
57	0	0	0	1	57	57	0	0
58	0	0	0	1	58	58	0	0
59	0	0	0	1	59	59	0	0
60	0	0	0	1	60	60	0	0
61	0	0	0	1	61	61	0	0
62	0	0	0	1	62	62	0	0
63	0	0	0	1	63	63	0	0
64	0	0	0	1	64	64	0	0
65	0	0	0	1	65	65	0	0
66	0	0	0	1	66	66	0	0
67	0	0	0	1	67	67	0	0
68	0	0	0	1	68	68	0	0
69	0	0	0	1	69	69	0	0
70	0	0	0	1	70	70	0	0
71	0	0	0	1	71	71	0	0
72	0	0	0	1	72	72	0	0
73	0	0	0	1	73	73	0	0
74	0	0	0	1	74	74	0	0
75	0	0	0	1	75	75	0	0
76	0	0	0	1	76	76	0	0
77	0	0	0	1	77	77	0	0
78	0	0	0	1	78	78	0	0
79	0	0	0	1	79	79	0	0
80	0	0	0	1	80	80	0	0
81	0	0	0	1	81	81	0	0
82	0	0	0	1	82	82	0	0
83	0	0	0	1	83	83	0	0
84	0	0	0	1	84	84	0	0
85	0	0	0	1	85	85	0	0
86	0	0	0	1	86	86	0	0
87	0	0	0	1	87	87	0	0
88	0	0	0	1	88	88	0	0
89	0	0	0	1	89	89	0	0
90	0	0	0	1	90	90	0	0
91	0	0	0	1	91	91	0	0
92	0	0	0	1	92	92	0	0
93	0	0	0	1	93	93	0	0
94	0	0	0	1	94	94	0	0
95	0	0	0	1	95	95	0	0
96	0	0	0	1	96	96	0	0
97	0	0	0	1	97	97	0	0
98	0	0	0	1	98	98	0	0
99	0	0	0	1	99	99	0	0
100	0	0	0	1	100	100	0	0

Measuring

Measuring Land with the Cross-Staff and Chain.

SUPPOSE *fig. 1 plate 4* to be a piece of land to be measured; take it across the middle, from *a* to *b*, 489 links; the length is 965 links, from *c* to *d*; multiply these together, and the product will be the content.

$$\begin{array}{r}
 965 \\
 \times 489 \\
 \hline
 8685 \\
 7720 \\
 3860 \\
 \hline
 471885 \\
 4 \\
 \hline
 287540 \\
 840 \\
 \hline
 3501600
 \end{array}$$

The content is 4 acres, 2 quarters, 35 poles. Always cut off 5 figures to the right-hand, and what is left, to the left-hand, is acres, in the first product; the remainder, multiplied by 4 gives quarters, and the last remainder multiplied by 40 gives poles.

Suppose a piece of land to be in the form of a trapezium, as *fig. 2 pl. 4*, the diagonal line *a b*, 1215 links, and the perpendiculars *c d* and *e f* 436 and 395. Take half the sum of the perpendiculars, and multiply that by the diagonal line, and the product will be the content.

$$\begin{array}{r}
 436 \\
 395 \\
 \hline
 831 \\
 \hline
 415 \text{ half sum.} \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 1215 \\
 415 \\
 \hline
 6075 \\
 1215 \\
 \hline
 4860 \\
 \hline
 \end{array}$$

the content in acres 5.04225

$$\begin{array}{r}
 4 \\
 \hline
 16900
 \end{array}$$

Suppose *fig. 3 pl. 4*, to be an irregular piece of land, as *a b c d e f*, to be divided into trapeziums and triangles. For the trapezium *a b e f* take half the sum of the perpendiculars *g b* and *f l*; multiply that by the base line

line ac , 945, and the product will be the content of the trapezium $abcf$; then, for the trapezium $efcd$, take half the sum of the perpendiculars db and if , multiply that by the whole base-line ec , and the product will be the content.

103	945 base-line ac .	212
90	186	160
193	5670	372
96 $\frac{1}{2}$ sum of db & if .	7560	186 $\frac{1}{2}$ sum of f & g b
	945	
460 base-line ce	1.75770	The whole content 2 acres
96	.44160	0 quarters 31 poles.
2760	2.19930	
4140	4	
44160	79720	
	40	
	31.188800	

In taking the dimensions in *fig. 3*, set the cross on the diagonal line ac , where it will catch the angles at f , and looking through the sights the other way to a or c , the line ac and lf will be at right angles. Then with the chain, take the dimensions; move the cross to g , and take the perpendicular gb , and so for any other.

To measure a piece of land, as *fig. 4*, *pl. 4*. whose boundaries lie in a curvilinear manner and the inner part a trapezium, divide the trapezium into two triangles, and take offsets at right-angles from the station-lines with the cross-staff, and add them together; divide the sum by the number of parts, and that gives a mean breadth; then multiply the mean breadth by the length of the station-line, and that will give the content of the out-limits, which add to the area or content of the two triangles, and that will be the content of the whole.

To measure the trapezium, *fig. 4, plate 4*, add the two perpendiculars together, and take half the sum; multiply that by the base-line, 1395, and it gives the content of the trapezium:

564
545

1109 Sum of the two perpendiculars.

554 Half sum of ditto, multiplied by

 [base-line. 1395 Links, the 554 Links.

Content of the trapezium 7 acres,
2 quarters, 36 poles.

5580

6975

6 975

Always point off 5 figures to the right-hand, and what remains to the left is acres; then multiply the remaining 5 figures by 4, and point off five, and what is to the left-hand is quarters; multiply the last remainder by 40, and point off five figures, and what is to the left-hand is poles.

7.72830

4

2.91320

40

36.52800

	Links.
Station-line of end <i>d</i>	572
	7
Off-sets 21 links.	4004
	4
Mean breadth 7 links.	16016
	40
Content 6 poles.	6.40640

	Links.
Station-line of end <i>c</i>	560
	9
Off-sets 27 links.	5040
	4
Mean breadth 9 links.	20160
	40
Content 8 poles.	8.06400

	Links.
Station-line of side <i>a</i>	985
	16
Off-sets 96 links.	5910
	985
Mean breadth 16 links.	15760
	4
Content 25 poles.	63040
	40
Content 25 poles.	25.21600

	Links.
Station-line of side <i>b</i>	964
	12
Off-sets 72 links.	11568
	4
Mean breadth 12 links.	46272
	40
Content 18 poles.	18.50880

7.2.36 Content of the trapezium or middle piece.

25 Content of side *a*.

18 Content of side *b*.

8 Content of end *c*.

6 Content of end *d*.

8.0.13 The whole content, 8 acres, 0 quarters, 13 poles.

Of GAUGING.

IN gauging, 282 solid or cube inches are a gallon of ale or beer, 231 cube inches a gallon of wine, 268.8 a gallon of corn measure, 2150.42 is a bushel, corn measure; these are divisors for squares. For circular areas the following divisors are to be used, 359.05 divisor for ale-gallons, 294.12 divisor for wine-gallons, 2738 divisor for corn bushels.

To find the area in ale or wine gallons, of any rectilinear plain figure, whether triangular, quadrangular, or multangular, suppose a back or cooler in the form of a parallelogram or long square to be 215 inches in length and 65.5 in breadth, what is the content or area in wine or ale gallons? Multiply 215 by 65.5, and the product will be the area in inches, which divide by 282 for ale, and 231 for wine, and the quotient will be the answer in gallons, at one inch deep.

$$\begin{array}{r}
 215 \\
 \times 65.5 \\
 \hline
 1075 \\
 1075 \\
 1290 \\
 \hline
 14082.5
 \end{array}$$

282) 14082.5 (49.9 gallons at one inch deep, provided the sides of the back stand at right angles with the bottom.

$$\begin{array}{r}
 282 \\
 \times 49.9 \\
 \hline
 2802 \\
 2538 \\
 \hline
 14082
 \end{array}$$

$$\begin{array}{r}
 2645 \\
 \times 2538 \\
 \hline
 107
 \end{array}$$

Note, If the back or cooler does not stand on a level basis, when the bottom is covered with liquor, take the depth in 8 or 10 places and add them together, and divide the sum by the number of places, and the quotient will be a mean depth, so that one place must be found in the cooler, to answer that depth, for a constant dipping-place.

294.12) 545660 (18.75 are in wine-gallons.

129412

51480

29412

220680

205884

147960

147060

.900

72

48

576

288

359.05) 3456.00 (9.64 are in ale-gallons.

323145

224550

215430

91200

71810

19390

To find the content of any prism, of what form soever its base is, in ale or wine gallons. First, find the solid content in inches; then divide that content by 282, for ale, and 231 for wine, the respective quotients will be the content in ale or wine gallons, at one inch in depth; or, if the area of the base, in gallons, be multiplied by the height of the tun in inches, withinside, the product will be the whole content in gallons.

To find the content of a tun, whose bases are alike and parallel, but less at top than at bottom, being the frustum of a pyramid. Suppose a tun, whose bases are parallelograms; the length of the greater base is 84 inches, and its breadth 60 inches, the length of the lesser base 54 inches, the breadth 42 inches, and the depth

of the tun 36 inches, the content in ale and wine gallons is required. Multiply 84 by 60, the product is 5040, the area of the greater base; then multiply 54 by 42, and the product is 2268, the area of the lesser base; then multiply the two areas together, and the product is 11430720, whose square root is 3380, a geometrical mean proportional.

area of the greater base	5040
area of the lesser base	2268
mean proportional	3380

10688

12 inches, a third of the depth.

21376
10688

282)128256(454.8 ale-gallons

To find the content of a tun whose bases are parallel and circular, being the frustum of a cone. To the rectangle of the two diameters add one third part of the square of the differences of the diameters, that sum is the square of a mean diameter, which, divided by 359.05 for ale, or by 294.12 for wine, gives the area of a mean circle in ale or wine gallons; which multiplied by the depth, gives the whole content.

Suppose the greater diameter 60 inches, the smaller diameter 48 inches, and the depth 30 inches, the content in ale and wine gallons is required. Multiply 60 by 48 and the product is 2880, to which add 48 (a third part of the square of the difference of the two diameters,) and the sum is 2928, the square of a mean diameter, which divide by 359.05, and the quotient will be gallons in the area of a mean circle, which multiplied by 30, the depth, the product will be 248.10 gallons, the content.

To find the drip or fall of a tun, suppose the tun so placed, that, when the bottom is but just covered on one side, the liquor is 4 inches deep on the other side; how much must be allowed for the fall of this tun? That is, how much liquor is there in the tun? The diameter, in the middle of four inches from the bottom is 61.6 inches, and the area answering thereto is 10.56, which

which multiply by two, (the half of four) the product is 21.136 gallons, and so much liquor will just cover the bottom at one inch deep.

Suppose the tun to be set so much on one side that the liquor just touches the top of the tun, and the liquor on the opposite side just cuts between the bottom and the staves, how much liquor will there be in the tun?

Square the bottom diameter, multiply that square by the top diameter, divide the last product by the sum of the two diameters, and to the quotient add the square of the bottom diameter, and divide the sum by 1077.15 for ale, or by 882.36 for wine; then multiply the quotient by the depth, and the product is the content in gallons.

Suppose the bottom diameter of the tun to be 48 inches, and the top diameter 38 inches; the square of 48 is 2304, which multiply by 38, and the product is 87552; divide this by 86 the sum of the two diameters, the quotient is 101.8, to which add 2304, the square of 48, and the sum will be 3322; this divided by 1077.15, the quotient is 3, which multiply by 36, the depth, and the product is 108, gallons of liquor in the tun.

When the frustum of a cone or pyramid is cut by a diagonal plain through the extremities of the diameter, (as the liquor in the tun represents,) such a solid is called a hoof.

If it be the hoof of a square frustum, instead of dividing by 1077.15, divide by 846 for ale, or by 693 for wine. The rest of the work is the same as above directed.

Fig. 6, pl. 4. To find the solid content of a square frustum, cut through the extremities of both bases by a diagonal plain, as *ab*, in *fig. 6*, in two parts, as *abc* and *adb*.

To explain the work of this figure, first find the solid content of the whole frustum, then find the solid content of each hoof, and add them together; the sums will be equal to the whole frustum.

16	16	16 rectangle of
12	12	12 [16 by 12
144 square of lesser base.	96	192 [base.
	16	256 sq. of great
	256 sq. of great base.	144 sq. of lesser
		[base.
		592 sum of all.
		3 $\frac{1}{3}$ of the
		[height.
Solid content of the whole frustum		1776

The work of the hoof.

256 square of great base.	
12 side of lesser base.	
sum of the two bases	28
3072	109.71
28	256 add square of great base
272	365.71 sum
257	3 a third of the height
300	1097.13 content of great hoof
196	
40	
144 square of lesser base	
16 side of great base	
224	144 add square of lesser base
64	226.385
36	
80	226.285
	3 a third of the height
	678.855 content of lesser hoof
	1097.13 add great hoof

content of the two hoofs equal to 1775.985 the frustum.

Fig. 2. pl. 4. Let a, b, c, d be a copper to be gauged; the diameter at the bottom 26 inches, the diameter at top 36 inches, and the depth 24 inches.

To

To find the content of the copper, from the crown to the top, take a diameter in the middle of every six inches of the depth, then find the area of each diameter, and multiply the areas by the depth, 6 inches, and that will give the content in inches to every 6 inches in depth; then, adding all the contents together, the sum will be the content of the whole from the crown. This whole content being divided by 282 for ale, or 231 for wine, the quotient will be gallons; or, if you square the diameter of each dimension and divide that square by 359.06 for ale, or by 204.12 for wine, the quotient will be gallons at one inch depth; then each area, in gallons multiplied by the depth, 6 inches, gives the content of every 6 inches in gallons, then add all the contents together, the sum will be the content of the whole, in gallons, exclusive of the crown.

To find the content of liquor that will just cover the crown, when the crown is just covered with liquor, let it off into a regular vessel, and gauge it, and add the content, the sum will be the whole content of the copper.

Now, if the crown of the copper be taken for the frustum of a sphere, the content must be found as in page 32

To compute the content of any close cask, the dimensions must be carefully taken, according to the following rules, see fig. 12, pl. 4.

viz. $\left\{ \begin{array}{l} \text{the bung diameter,} \\ \text{the head diameter,} \\ \text{the length of the cask,} \end{array} \right\}$ within the cask.

In taking the dimensions it must be observed, that the bung-hole be in the middle of the cask; also that the bung-staff and the staff opposite to the bung-hole be regular and even within, and that the heads of the cask be equal and truly circular; if so, the distance between the inside of the chine to the outside of its opposite staff will be the head-diameter within the cask very nearly; and for the length of the cask take to the outside of the two heads; from that length subtract $1\frac{1}{2}$ inch, more or less,

as you may judge, for the thickness of the heads; the remainder will be the length of the cask within.

To find the content of the cask, multiply the difference of the bung and head diameters by .65, and add the product to the head-diameter; and the sum is a mean diameter.

Suppose the bung-diameter 30 inches, the head-diameter 24 inches, and the length 42 inches. The difference between the bung and head diameter is 6, which multiply by .65 and the product is 3.90, which added to the head-diameter the sum is 27.90, the mean diameter: the area answering thereto is 2.16 ale gallons, which multiplied by the length gives 90.72 gallons, the whole content of the cask. By finding a mean diameter the cask is reduced to a cylinder. See the work.

$$\begin{array}{r}
 .65 \\
 6 \\
 \hline
 3.90 \\
 24 \\
 \hline
 27.90 \text{ mean diameter} \\
 27.90 \\
 \hline
 251100 \\
 19510 \\
 5580 \\
 \hline
 359.05778.4100(\quad 2.16 \text{ ale-gallons} \\
 71810 \quad 42 \text{ length} \\
 \hline
 60310 \quad 432 \\
 35905 \quad 864 \\
 \hline
 244050 \quad 90.72 \text{ the whole content in ale-} \\
 215430 \quad \text{gallons} \\
 \hline
 28620
 \end{array}$$

MENSURATION

MENSURATION OF SUPERFICIAL FIGURES, LAND, &c.

Of a Parabola. Fig. 5, Plate 4.

A parabola is a curvilinear figure, being the section of a cone cut by a plain parallel to one of its sides. Every parabola is two thirds of its circumscribing parallelogram, therefore, to find the area, multiply the base-line, ab , by the perpendicular height, dc ; then multiply that product by 2, and divide the product by 3, the quotient will be the area of the parabola. See the work,

26.5 base-line, ab .

18.3 perpendicular, cd .

295

2130

4260

414.95

2

3)969.90

323.30 area of the parabola.

To measure the Segment of an Ellipsis. Fig. 10, Plate 4.

All segments of ellipses and the inscribed circle, whose base or chord line is parallel to the transverse diameter, and which have the same height, are in proportion one to another as the area of the ellipsis and circle is, that is, as the area of the circle is to the area of the ellipsis, so is the segment $b b b$ to the segment $a b c$; or, as $g h$ is to $e f$ so is the area of the segment $b b b$ to the area of the segment $a b c$.

Suppose the diameter of the great circle, $e f$, to be 60, the conjugate diameter, $g h$, to be 45, and the area of the segment, $b b b$, 119.38; then, as 45 is to 119.38 so is 60 to the area of the segment, $a b c$.

((74))

$$45 : 119.38 :: 60$$

$$45 \overline{) 7162.80} (159.17 \text{ area of segment } a b c.$$

45

266

285

412

405

78

45

330

315

15

Suppose the area of the inscribed circle, in *fig. 10*, to be 1590.43, the area of the ellipsis to be 2120.58, and the area of the segment of the inscribed circle, *b b b*, to be 119.38 then will it be

Table of Divisors.

$$\text{as } 1590.43 : 2120.58 :: 119.38$$

1	1590.43
2	3180.86
3	4771.29
4	6361.72
5	7952.15
6	9542.58
7	11133.01
8	12723.44
9	14313.87

11938

1696464

636174

1008522

212058

212058

$$159.43 \overline{) 2120.58} = 13.32$$

area of *a b c* ellipsis segment as before.

Fig. 11, Plate 4. To find the area or superficial content of the segment of an ellipsis, whose base or chord-line is parallel to the conjugate diameter, as *g b i* in *fig. 11*.

If you circumscribe any ellipsis with a circle, and suppose a chord-line to be drawn therein parallel to the conjugate diameter, as *g b i*, then it will be as *o g*, the diameter of the circle, is to *a c*, the conjugate diameter of

of the ellipsis, so is $y b z$, the chord-line of the circular segment to $g b i$ the chord-line of the elliptic segment; that is, all segments of ellipses and their circumscribing circles, whose bases are parallel to the conjugate diameter, and of the same height, are in proportion one to another as their bases are, that is, as $y b z$ is to $g b i$ so is the area of the segment $y b z$ to the area of the segment $g b i$; or, as $d b$, the diameter of the circle, is to $a c$, the conjugate diameter of the ellipsis, so is the area of the segment $y b z$ to the area of the segment $g b i$. Suppose the base-line of the circle's segment, $y b z$, to be 36, and the base-line of the elliptic segment $g b i$, 28, and the area of the circle's segment 146.7, then it will be as 36 is to 28 so is 146.7 to the area of the ellipsis. See this work.

$$\begin{array}{r}
 36 : 28 :: 146.7 \\
 \hline
 11236 \\
 2934 \\
 \hline
 36 \overline{) 4107.6} \\
 \hline
 114.1 \text{ area of elliptic segment.}
 \end{array}$$

The area of every ellipsis is a mean proportional between the areas of its circumscribing and inscribing circles. That is, as the greater circle's area is to the area of the ellipsis so is the area of the ellipsis to the area of the lesser circle.

To find the superficial Content of a Square Frustum, as Fig. 6, Plate 4.

Add the two bases together, 16 inches and 12 inches; take half the sum, and multiply that by 9 feet, the height, then multiply that product by 4, the number of sides, and divide by 12; the quotient will be the answer in feet, to which add the area of the two bases, which will be the whole superficial content.

To find the superficial Content of a rectangular Pyramid, whose Bases are parallel one to another but disproportional, as Fig. 7, Plate 4.

Take half the perimeter of the greater base, which is 48, and half the perimeter of the lesser base, which is 30, add them together, the sum is 78, which multiply by 6, the height, the product is 468, which divide by 12, the quotient is 39 feet; to which add the sum of the two bases, 5 feet, the sum is 44 feet, the whole superficial content.

To find the superficial Content of ^{the frustum of} a Cone, having its Bases parallel to each other, but unlike. Fig. 8, Plate 4.

To the circumference of the ellipsis 94.52 add the circumference of the circle 75.42, the sum is 169.94, and the half thereof 84.97, which multiply by 6, the height, the product is 509.82, which divide by 12, the quotient is 42.485 feet the curve surface, then the area of the ellipsis, or lower base, is 4.71 feet, and the area or upper base 3.12 feet; add them to the curve surface, the sum is 50.31 feet, the whole superficial content.

To find the superficial Content of a Cone. Fig. 5, Plate B.

Multiply the circumference by the slant height, and the product is the curve surface; to which add the area of the base, and that will be the whole superficial content.

To find the superficial Content of a hexangular Pyramid, Fig. 4, Plate B.

Multiply half the sum of the sides by the height, taken from the middle of one of the sides.

54, half sum of the sides,	
15 height	
270	
54	
1210	
67.5	content of curve surface.

To measure the Frustum of a rectangular Pyramid, whose Bases are parallel one to another, but disproportional.

Fig. 7, Plate 4.

A b c d is the greater base, and *e f g h* the lesser base; the greater base 30 inches by 18, the lesser base 21 inches by 6, and the height 6 feet, the solid content is required. To 30, the length of the greater base, add 10.5, half the lesser base, the sum will be 40.5; which multiply by 18, the greater breadth, and the product then is 729.0, which reserve. To the length of the lesser base, 21, add 15, which is half the greater base, the sum will be 36, which multiply by 9, the lesser breadth, the product is 324, to which add 729.0 the reserved product, and the sum is 1053.0, which multiply by 2, a third part of the height, the product will be 2106.0. Divide this product by 12, and the quotient arising therefrom divide by 12, the last quotient will be the answer, or solid content, in feet and parts of a foot. See the work.

30 length of greater base.	21 lesser base
10.5 half the lesser base.	15 half greater base added.
40.5	36 sum.
18 greater base in width.	9 lesser breadth.
3240	324
405	729.0 reserved product added.
729.0 reserved product.	1053.0
	2 a third part of the height.
	12) 2106.0
	12) 175500
	14.625 solid content.

To measure the Frustum of a Cone, having its Bases parallel to each other, but unlike. Fig. 8, Plate 4.

A b c d is the frustum of a cone, whose bottom base is an oval, the transverse diameter *a b* being 36 inches, the conjugate diameter *e f* 24 inches, and the height of the

the frustum 6 feet; the solid content is required. To 36, the diameter of the greater base $a b$, add 12, half the diameter of the lesser base, the sum will be 48, which multiply by 24, the conjugate diameter of the great base $f e$, the product will be 1152, the reserved product. Then, to 24, the diameter of the lesser base, add 18, half the transverse diameter of the greater base, and the sum is 42, which multiply by 24, the diameter of the lesser base, the product will be 1008, to which add the reserved product, 1152, the sum will be 2160, which multiply by 7854, the product will be 696.4640; which multiply by 2, a third part of the height, the product will be 3392.9280, which divide by 12, the quotient is 28.2746, which divide again by 12 the quotient is 2.3562, the solid content, in feet and parts of a foot.

To measure a parabolic Conoid. Fig. 9, Plate 4.

A parabolic conoid is something like half a spheroid, having its sides straiter. It is generated by supposing a semi-parabola turned about its axis. To find the solid content, the rule is, multiply the square of the diameter of the base by 7854, then multiply that product by half the height, and the last product will be the solid content; or, if you multiply by the whole height and take half the product, it will be the same thing, and divide by 1728. Let $a b c d$ be a parabolic conoid, the diameter of the base $a b$ 30 inches, and the height $c d$ 26 inches.

30		7854 tabular number	
30		900 square of base	
900 square of the bases		706.8600	
		27 inches high	
		49480200	
		14137200	
		2)190852900	
		95426100	
		5.5223 solid content in feet and	
		[parts of a foot.	

Fig. 9, Plate 4. Supposing the parabolic conoid to be cut off by a plain at ef parallel to the base-line, ab , to find the solid content of the frustum, $abef$, this is the rule; multiply the sum of the square of the two bases by .3927, and that product by the height of the frustum, the last product will be the solid content. Suppose the base ab 30 inches, the upper base ef 20 inches, and the height of the frustum 16 inches, the solid content is required.

$\begin{array}{r} 20 \\ 20 \\ \hline 400 \end{array}$	$\begin{array}{r} 30 \\ 30 \\ \hline 900 \end{array}$
square of ef .	square of ab ,
400	900

$$\begin{array}{r} 1300 \text{ sum of both} \\ .3927 \text{ tabular number.} \\ \hline 1300 \text{ sum of two bases} \end{array}$$

1178100

3927

5105100

.16 height of the frustum

30630600

5105100

$$\begin{array}{r} 1728 \text{ ft} \text{ pns of a ft} \\ 81681600 (4 \cdot 726) \\ \hline 6912 \end{array}$$

12561

02096

4656

3456

12000

10368

1632

To measure a Spheroid. Fig. 10, Plate 4.

A spheroid is a solid resembling an egg; to find the solid content thereof, the rule is, multiply the square of the diameter, gh , 26 inches, by the length, ef , 48 inches, and that product being multiplied by .5236, the last product will be the solid content.

To find the solid Content of a Spheroid, as $a b c d$ Plate 4. Fig. 11, by the Rotation of the Semi-Ellipsis, $a b c d$, about its transverse Axis.

Let $d b$, the length of the spheroid, be 21.20 inches, (which is the diameter of the circumscribing circle, or sphere,) and $a c$, the diameter of the greatest circle of the spheroid, 14 inches; the solid content is required.

$$\begin{array}{r}
 14 \\
 14 \\
 \hline
 56 \\
 14 \\
 \hline
 196 \text{ square of } a c \\
 21.20 \\
 \hline
 3920 \\
 196 \\
 \hline
 392 \\
 4155.20 \\
 .5236 \\
 \hline
 2493120 \\
 1246560 \\
 831040 \\
 2077600 \\
 \hline
 2175.662720 \text{ solid content of the spheroid.}
 \end{array}$$

Then it will be, as 4988.92782, the solid content of the sphere, is to 2175.718720, the solid content of the spheroid, so is 701.624, the solid content of the frustum of the sphere, to 305.9848, the solid content of the frustum of the spheroid. See the work.

solid content of the sphere.	—	solid content of the spheroid.	—	solid content of the frustum of the sphere
4988.927		2175.718		701.624
		701.624		

$$\begin{array}{r}
 8702872 \\
 4351436 \\
 13054308 \\
 2175718 \\
 152300260 \\
 \hline
 4988 \ 927 \ 15265359.663032 \text{ (the frustum of the sphere)} \\
 305.984 \text{ solid content of}
 \end{array}$$

4988927) 1526536.966032(305.984

1496678

10858866

24944635

49144310

44900343

42419673

39911416

25082572

19955708

5126864

To find the solid Content of the Middle-Zone. Fig. 11, Pl. 4.

The proportion holds good for finding the solid content of the middle zone; for, as the solid content of the whole sphere is to the solid content of the whole spheroid, so is the middle zone of the sphere to the middle zone of the spheroid, having the same height as the middle zone of the sphere, for eyz h is equal in height to the middle zone of the spheroid gip , and so for any other. From the solid or whole content of the sphere subtract the solid content of the two frustums, and the remainder will be the solid content of the middle zone.

701.624 solid content of the frustum ydz

2

1403.248 solid content of the two frustums.

solid content of the frustum of the spheroid 305.984

2

content of the two frustums of the spheroid 611.968

2175.718 solid content of spheroid

611.968

1563.750 solid content of the middle-zone of the spheroid

solid content of the sphere 4988.927

content of the two frustams 1403.248

content of the middle zone of 3585.679 the sphere

To find the Periphery or Circumference of the Ellipsis.

To twice the square root of the sum of the squares of the transverse and conjugate diameters add one third part of the conjugate diameter, and the sum will be the circumference or periphery of the ellipsis. Let the transverse diameter be 12 feet and the conjugate diameter 8 feet.

$$\begin{array}{r}
 8 \\
 8 \\
 \hline
 64 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 12 \\
 12 \\
 \hline
 144 \\
 64 \\
 \hline
 208
 \end{array}$$

208 (14.42 square root)

$$\begin{array}{r}
 14.42 \\
 2 \\
 \hline
 28.84 \\
 2.5 \text{ add} \\
 \hline
 31.34 \text{ periphery or circumference of the ellipsis}
 \end{array}$$

Suppose the transverse diameter to be 20 feet and the conjugate diameter 15 feet.

$$\begin{array}{r}
 20 \\
 20 \\
 \hline
 400 \\
 225 \\
 \hline
 625
 \end{array}
 \qquad
 \begin{array}{r}
 15 \\
 15 \\
 \hline
 225 \\
 15 \\
 \hline
 240
 \end{array}$$

square of transverse diam. 400

sum of both 625

sq. of conjugate diam. 225

625 (25 square root)

45 (225 square root)

50 twice the root

5 add a third of the conjugate diameter

55 circumference

Of a Triangle,

Whose three sides are as follow, viz. one side 36 feet 5 tenths, another side 28 feet 2 tenths, and the third side 21 feet 3 tenths. To find the superficial content, without the help of a perpendicular, add the three sides together, and take half the sum; then subtract each side from that half sum, which done, multiply the half sum and the three differences continually, and out of the last product extract the square root, which square root will be the area or superficial content of the triangle.

36.5	43.0	43.0	43.0
28.2	36.5	28.2	21.3
21.3	6.5	14.8	21.7
sum of the sides 86.0	the three differences		
43.0			
6.5			
2150	89547.22. (29.92		
2580	4		
279.50	42)495		
14.8	441		
223600	589)5447		
110800	5301		
27950	5982)14622		
4126.600	11964		
21.7	2658		
28886200			
4126600			
8253200			
89547.2200			

The aliquot Parts of a Foot. To be learnt by Heart.

Two inches one sixth.

Three inches one fourth.

Four inches one third.

Five inches one third and one twelfth.

Six inches one half.

Seven inches one third and one fourth.

Eight inches two thirds.

Nine inches one half and one fourth.

Ten inches one half and one third.

Eleven inches one half, one third, and a twelfth.

l
 c

a

ab

o

d

Plate IV to face the last page

